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Abstract

The paper is devoted to discretization of integral norms of functions from a given collection of finite dimensional subspaces. For natural collections of subspaces of the multivariate trigonometric polynomials we construct sets of points, which are optimally (in the sense of order) good for each subspace of a collection from the point of view of the integral norm discretization. We call such sets universal. Our construction of the universal sets is based on deep results on existence of special nets, known as (t, r, d)-nets.

1 Introduction

This paper is a follow up to the two recent papers [31] and [32]. As it is clear from the title the two main concepts of the paper are *discretization* and *universality*. Discretization is a very important step in making a continuous problem computationally feasible. The problem of construction of good sets of points in a multidimensional domain is a fundamental problem of mathematics and computational mathematics. A prominent example of a classical discretization problem is the problem of metric entropy (covering numbers, entropy numbers). The reader can find fundamental general results on metric entropy in [16], Ch.15, [29], Ch.3, [4], [23] and in recent papers [30], [9]. Bounds for the entropy numbers of function classes are important by themselves and also have important connections to other fundamental problems (see, for instance, [29], Ch.3 and [5], Ch.6). Another prominent example of a discretization problem is the problem of numerical integration. Numerical

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