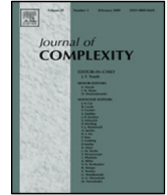




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Covering and separation of Chebyshev points for non-integrable Riesz potentials[☆]

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ABSTRACT

For Riesz s -potentials $K(x, y) = |x - y|^{-s}$, $s > 0$, we investigate separation and covering properties of N -point configurations $\omega_N^* = \{x_1, \dots, x_N\}$ on a d -dimensional compact set $A \subset \mathbb{R}^d$ for which the minimum of $\sum_{j=1}^N K(x, x_j)$ is maximal. Such configurations are called N -point optimal Riesz s -polarization (or Chebyshev) configurations. For a large class of d -dimensional sets A we show that for $s > d$ the configurations ω_N^* have the optimal order of covering. Furthermore, for these sets we investigate the asymptotics as $N \rightarrow \infty$ of the best covering constant. For these purposes we compare best-covering configurations with optimal Riesz s -polarization configurations and determine the s th root asymptotic behavior (as $s \rightarrow \infty$) of the maximal s -polarization constants. In addition, we introduce the notion of “weak separation” for point configurations and prove this property for optimal Riesz s -polarization configurations on A for $s > \dim(A)$, and for $d - 1 \leq s < d$ on the sphere S^d .

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1. Introduction

Suppose A is a compact subset of a Euclidean space \mathbb{R}^d and $\omega_N = \{x_1, \dots, x_N\} \subset A$ is a *multiset* (or an N -point configuration); i.e., a set of points with possible repetitions and cardinality $\#\omega_N = N$,

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counting multiplicities. For a positive number s we put

$$P_s(A; \omega_N) := \inf_{y \in A} \sum_{j=1}^N \frac{1}{|y - x_j|^s}.$$

Then the N th s -polarization (or Chebyshev) constant of A is defined by

$$\mathcal{P}_s(A; N) := \sup_{\omega_N \subset A} P_s(A; \omega_N).$$

We note that since A is compact, there exists for each $N \in \mathbb{N}$ a configuration $\omega_N^* = \{x_1^*, \dots, x_N^*\}$ and a point y^* such that

$$\mathcal{P}_s(A; N) = P_s(A; \omega_N^*) = \sum_{j=1}^N \frac{1}{|y^* - x_j^*|^s}. \tag{1}$$

We call ω_N^* an *optimal (or extremal) Riesz s -polarization configuration* or simply an *optimal configuration*.

From an applications prospective, the maximal polarization problem, say on a compact surface (or body), can be viewed as the problem of determining the smallest number of sources (injectors) of a substance together with their optimal locations that can provide a required saturation of the substance at every point of the surface (body).

The general notion of polarization (or Chebyshev constants) for potentials was likely first introduced by Ohtsuka [18]. Further investigations of the asymptotic behavior as $N \rightarrow \infty$ of polarization constants as well as the asymptotic behavior of optimal configurations appear, for example, in [1,8,10,9,2,20,4,3,19].

The following result is a special case of a theorem due to Borodachov, Hardin, Reznikov and Saff [3] (see also [4]). It describes the asymptotic behavior of optimal configurations for the case of non-integrable Riesz kernels on A . Here and throughout we denote by \mathcal{H}_d the Hausdorff measure on \mathbb{R}^ℓ , $d \leq \ell$, normalized by $\mathcal{H}_d([0, 1]^d) = 1$, where $[0, 1]^d$ is the d -dimensional unit cube embedded in \mathbb{R}^ℓ .

Theorem 1.1. *Suppose A is a compact C^1 -smooth d -dimensional manifold, embedded in \mathbb{R}^ℓ with $d \leq \ell$, and $\mathcal{H}_d(\partial A) = 0$, where ∂A denotes the boundary of A . If $s > d$, then there exists a positive finite constant $\sigma_{s,d}$ that does not depend on A such that*

$$\lim_{N \rightarrow \infty} \frac{\mathcal{P}_s(A; N)}{N^{s/d}} = \frac{\sigma_{s,d}}{\mathcal{H}_d(A)^{s/d}}. \tag{2}$$

Moreover, if $\{\omega_N^*\}_{N=1}^\infty$ is any sequence of optimal configurations satisfying (1), then the normalized counting measures μ_N^* for the multisets ω_N^* satisfy

$$\mu_N^* := \frac{1}{N} \sum_{x \in \omega_N^*} \delta_x \xrightarrow{*} \mu,$$

where $\xrightarrow{*}$ denotes convergence in the weak* topology, and μ is the uniform measure on A ; i.e., for any Borel set $B \subset \mathbb{R}^\ell$

$$\mu(B) = \frac{\mathcal{H}_d(B \cap A)}{\mathcal{H}_d(A)}.$$

In other words, in the limit, optimal polarization configurations ω_N^* for non-integrable Riesz potentials are uniformly distributed in the weak* sense. In this paper we study more distributional properties of optimal configurations ω_N^* . In particular, we investigate their separation, their covering (or mesh) radius, and their connection to the “best covering problem” for the set A .

Definition 1.2. Let A be a compact subset of a Euclidean space. For any N -point configuration $\omega_N \subset A$, the *separation constant* of ω_N is defined by

$$\delta(\omega_N) := \min_{i \neq j} |x_i - x_j|$$

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