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Tractability properties of the weighted star discrepancy of regular grids

Friedrich Pillichshammer*

Abstract

In this paper we study tractability properties of the weighted star discrepancy with general coefficients of centered regular grids with different mesh-sizes. We give exact characterizations of the weight sequences $(\gamma_j)_{j\geq 1}$ such that the regular grid with different mesh-sizes achieves weak, uniform weak, quasi polynomial, polynomial or strong polynomial tractability for the $\boldsymbol{\gamma}$-weighted star discrepancy. For example, a necessary and sufficient condition such that the regular grid with different mesh-sizes achieves weak tractability for the $\boldsymbol{\gamma}$-weighted star discrepancy is $\lim_{j\to\infty} j\gamma_j = 0$.

Keywords: weighted star discrepancy, tractability, regular grid, quasi-Monte Carlo
MSC 2010: 11K38, 11K45, 65C05

1 Introduction

In this short paper we study the weighted star discrepancy of the *centered regular grid* with different mesh-sizes $m_1,\ldots,m_d$ in dimension d, given by

$$\Gamma_{m_1,\ldots,m_d} = \left\{ \left(\frac{2\ell_1+1}{2m_1},\ldots,\frac{2\ell_d+1}{2m_d} \right) \ : \ \ell_j \in \{0,1,\ldots,m_j-1\} \text{ for } j=1,\ldots,d \right\},$$

where $m_1,m_2,\ldots,m_d \in \mathbb{N}$ are the mesh-sizes for the single coordinate directions.

It may make one wonder why the discrepancy of regular grids is worth to be studied since it is well known that the (classical, i.e., unweighted) star discrepancy of regular grids is far from being of optimal order (see, e.g. [9, Remark 2.20]). However, in several recent papers it turned out, that the use of regular grids with different mesh-sizes in the context of numerical integration and approximation in weighted spaces of infinitely smooth functions leads to optimal results (e.g., exponential convergence rates or various notions of tractability); see [3, 4, 8]. These findings motivate the study of the weighted star discrepancy of regular grids. Clearly, in the unweighted case the star discrepancy of

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