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(s, t)-weak tractability of Euler and Wiener integrated processes ☆

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ABSTRACT

We study (s, t)-weak tractability of function approximation in the average case setting with respect to a zero-mean Gaussian measure. A problem is (s, t)-weakly tractable for positive s and t if its information complexity is not an exponential function of the sth power of the reciprocal of the accuracy ε and the tth power of the number d of variables. We give necessary and sufficient conditions on the (s, t)-weak tractability of the approximation of multivariate Euler and Wiener integrated processes.

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1. Introduction

Tractability of multivariate problems $S = \{S_d\}_{d\in\mathbb{N}}$ is concerned with the behavior of the information complexity $n(\varepsilon, S_d)$ when the accuracy ε of approximation goes to zero and the number d of variables goes to infinity. Information complexity is defined as the minimal number of linear functionals needed to find an ε-approximation of $S_d : F_d \to G_d$. In this paper we study the average case setting with a zero-mean Gaussian measure μ_d, for the normalized error criterion. More precisely, F_d is a separable Banach space equipped with a zero-mean Gaussian measure μ_d, G_d is a Hilbert space, and an algorithm $A : F_d \to G_d$ is said to be an ε-approximation of S_d if

$$\left(\int_{F_d} \|S_d(f) - A(f)\|^2_{G_d} \mu_d(df)\right)^{1/2} \le \varepsilon\left(\int_{F_d} \|S_d(f)\|^2_{G_d} \mu_d(df)\right)^{1/2}.$$

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Various notions of tractability have been studied recently for many multivariate problems. The two most important and widely studied problems are approximation and integration of multivariable functions. We briefly recall some of the basic tractability notions.

As in [9], we say that S is

- *strongly polynomially tractable (SPT)* iff there are non-negative numbers C and p such that

$$n(\varepsilon, S_d) \le C\,\varepsilon^{-p} \quad \text{for all} \quad \varepsilon \in (0,1),\ d \in \mathbb{N};$$

- *polynomially tractable (PT)* iff there are non-negative numbers C, p and q such that

$$n(\varepsilon, S_d) \le C\,\varepsilon^{-p}\, d^{q} \quad \text{for all} \quad \varepsilon \in (0,1),\ d \in \mathbb{N}.$$

As in [2,5,6,8] respectively, we say that S is

- *quasi-polynomially tractable (QPT)* iff there are non-negative numbers C and t such that

$$n(\varepsilon, S_d) \le C\, \exp\big(t\,(1+\ln \varepsilon^{-1})(1+\ln d)\big) \quad \text{for all} \quad \varepsilon \in (0,1),\ d \in \mathbb{N};$$

- *uniformly weakly tractable (UWT)* iff

$$\lim_{\varepsilon^{-1}+d\to\infty} \frac{\ln\, n(\varepsilon, S_d)}{\varepsilon^{-\alpha} + d^{\beta}} = 0 \quad \text{for all} \quad \alpha, \beta > 0;$$

- *weakly tractable (WT)* iff

$$\lim_{\varepsilon^{-1}+d\to\infty} \frac{\ln\, n(\varepsilon, S_d)}{\varepsilon^{-1} + d} = 0;$$

- *(s,t)-weakly tractable ((s,t)-WT)* for some $s, t > 0$ iff

$$\lim_{\varepsilon^{-1}+d\to\infty} \frac{\ln\, n(\varepsilon, S_d)}{\varepsilon^{-s} + d^{t}} = 0.$$

SPT, PT, QPT and WT of the approximation of Euler and Wiener integrated processes have been studied in [4], while UWT in [7]. For these approximation problems we have $F_d = C([0,1]^d)$ and $G_d = L^2([0,1]^d)$. The measures μ_d for Euler and Wiener integrated processes are defined in terms of the nondecreasing sequence $\{r_k\}_{k\in\mathbb{N}}$ of nonnegative integers

$$0 \le r_1 \le r_2 \le r_3 \le \cdots .$$

Roughly speaking, r_k measures the smoothness of the process with respect to the kth variable.

The following conditions have been obtained therein:

- For the Euler integrated process:

$$\begin{aligned}
WT &\Leftrightarrow \lim_{k\to\infty} r_k = \infty,\\
UWT &\Leftrightarrow \liminf_{k\to\infty} \frac{r_k}{\ln k} \ge \frac{1}{2\ln 3},\\
QPT &\Leftrightarrow \sup_{d\in\mathbb{N}} \frac{\sum_{k=1}^{d}(1+r_k)3^{-2r_k}}{\max(1,\ln d)} < \infty,\\
PT \Leftrightarrow SPT &\Leftrightarrow \liminf_{k\to\infty} \frac{r_k}{\ln k} > \frac{1}{2\ln 3}.
\end{aligned}$$

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