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A topological view on algebraic computation models

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Abstract

We investigate the topological aspects of some algebraic computation models, in particular the BSS-model. Our results can be seen as bounds on how different BSS-computability and computability in the sense of computable analysis can be. The framework for this is Weihrauch reducibility. As a consequence of our characterizations, we establish that the solvability complexity index is (mostly) independent of the computational model, and that there thus is common ground in the study of non-computability between the BSS and TTE setting.

Keywords: Weihrauch reducibility, BSS-machine, Analytic machine, Effective DST, solvability complexity index, TTE, Computable analysis
2010 MSC: 03D78, 68Q05, 12Y05

1. Introduction

There are two major paradigms for computability on functions on the real numbers: On the one hand, computable analysis in the tradition of Grzegorczyk [1, 2] and Lacombe [3] as championed by Weihrauch [4, 5] (see also the equivalent approaches by Pour-El and Richards [6] or Ko [7]). On the other hand, the BSS-machines by Blum, Shub and Smale [8, 9], or the very similar real-RAM model. Incidentally, both schools claim to be in the tradition of Turing.

Computable analysis can, to a large extent, be understood as effective topology [10, 11] – this becomes particularly clear when one moves beyond just the real numbers, and is interested in computability on spaces of subsets or functionals. In particular, we find that the effective Borel hierarchy occupies the position analogous to the arithmetical hierarchy in classical recursion theory; and that incomputability of natural problems is typically a consequence of dis-

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