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Abstract

In this paper we investigate a special sequence related to the Halton sequence, namely the Halton sequence indexed by $\lfloor n\beta \rfloor$ with $\beta \in \mathbb{R}$, and prove a metric almost low-discrepancy result.

Keywords: Halton sequences, Subsequences, Discrepancy.
2000 MSC: 11K31, 11K38.

1. Introduction

The *discrepancy* of the first N terms of any sequence $(\boldsymbol{z}_n)_{n\geq 0}$ of points in $[0,1)^s$ is defined by

$$D_N(\boldsymbol{z}_n) = \sup_J \left| \frac{A_N(J)}{N} - \lambda_s(J) \right|,$$

where the supremum is extended over all half-open subintervals J of $[0,1)^s$, λ_s denotes the s-dimensional Lebesgue measure, and the counting function $A_N(J)$ is given by

$$A_N(J) = \#\{0 \leq n \leq N-1 \ : \ \boldsymbol{z}_n \in J\}.$$

For the sake of simplicity, we will sometimes write D_N instead of $D_N(\boldsymbol{z}_n)$. If the supremum is extended over all half-open subintervals J of $[0,1)^s$ with the lower left point in the origin then we arrive at the notion of the *star discrepancy* D_N^*. It is not so hard to see that $D_N^* \leq D_N \leq 2^s D_N^*$. A sequence $(\boldsymbol{z}_n)_{n\geq 0}$ of points in $[0,1)^s$ is called uniformly distributed if $\lim_{N\to\infty} D_N = 0$. (We refer the interested reader to (5) for a detailed background on the theory of uniform distribution.)