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Stabilizing effect of the power law inflation on isentropic relativistic fluids

Changhua Wei

Department of Mathematics, Zhejiang Sci-Tech University, Hangzhou, 310018, China

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Abstract

This paper is concerned with the stabilizing effect of the power law inflation on relativistic Eulerian fluids. We prove the global stability of the background solutions to the relativistic fluids including the isothermal gases and generalized Chaplygin gases by the method of conformal transformation when the initial data is a small perturbation to the background solution. We also prove the blowup phenomena of the relativistic Euler fluids including the isothermal gases and polytropic gases when the initial data satisfies suitable assumptions.

Keywords: Relativistic Euler equations; Power law inflation; Conformal transformation

1. Introduction

In this paper, we study the global stability and blowup phenomena of classical solutions to the following $1+3$-dimensional relativistic Euler equations in standard coordinates (t, x^1, x^2, x^3)

$$\nabla_\mu T^{\mu\nu} = 0, \tag{1.1}$$

where Greek indice μ takes its values in $\{0, 1, \cdots, 3\}$, ∇_μ denotes the covariant derivative with respect to the given metric $g = (g_{\mu\nu})$ and $T^{\mu\nu}$ denotes the energy momentum tensor, whose

E-mail address: changhuawei1986@gmail.com.

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components are given by

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu + pg^{\mu\nu},$$

in which ρ represents the energy density, $p = p(\rho)$ denotes the fluid pressure. $u = (u^0, u^1, u^2, u^3)$ is the four-velocity, which is future directed ($u^0 > 0$) and timelike vector field normalized by

$$u_\mu u^\mu = g_{\mu\nu} u^\mu u^\nu = -1.$$

Throughout the whole paper, repeated upper and lower indices are always summed over their ranges and the index is raised and lowered by the following given metric

$$g = -dt^2 + a^2(t) \sum_{i=1}^{3} (dx^i)^2, \tag{1.2}$$

where $a(t) > 0$ is known as the scale factor. Spacetime metric of form (1.2) is important, since the experimental results indicate that our universe is accelerated expanding and approximately spacial flat. We say the spacetime is accelerated expanding, we mean

$$\frac{d}{dt} a(t) > 0 \quad \text{and} \quad \frac{d^2}{dt^2} a(t) > 0.$$

As is known to all, by the hyperbolic nature of relativistic Euler equations, the classical solution to (1.1) always blowup in finite time no matter how small and smooth the initial data is, especially in Minkowski spacetime. Indeed, so many results have been obtained to study the blowup phenomena of the initial value problem of system (1.1). A breakthrough has been made by Christodoulou [2], in which he considered the compressible and irrotational relativistic Euler equations with constant entropy. By combining the techniques of geometry and analysis, Christodoulou got the mechanism of the formation of singularities and furthermore constructed the weak solution (shocks) with Lisibach [3] under the assumption of spherical symmetry. Recently, his method has been investigated deeply and applied to various models, see his work with Miao [4] for classical Euler equations, Miao and Yu [13] for a single variational wave equation, Holzegel, Klainerman, Speck and Wong [8] for more general quasilinear wave equations. Besides, there is another way to study the formation of singularities for hyperbolic systems in several space variables. This method is firstly found by Sideris [19], who introduced some averaged quantities arising from the conservation law and studied the evolution equations satisfied by these quantities. Under appropriate assumptions on the fluids and the initial data, he can obtain a class of solutions, which will blowup in finite time. Sideris's method is also widely applied to various models. One can refer to [10,17,23] for classical fluids, [20,25,26] for nonlinear wave equations, [7] and [16] for relativistic fluids.

The results introduced above are on the blowup phenomena of the hyperbolic systems. However, there also exist some other results concerning the global stability of the hyperbolic flows. To the author's best knowledge, there are mainly two structures to ensure the global stability of hyperbolic systems. The first one is the "linearly degenerate" characteristics, and this structure is satisfied by the Eulerian flows such as Chaplygin gas and stiff matter. Based on this structure, Godin [6] proved the global stability of 3D radial solutions to Chaplygin gases with variable entropy. His result is extended to 2D case by Ding, Witt and Yin [5]. By different method, Lei and