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J. Differential Equations 265 (2018) 3618–3649

Journal of Differential Equations

www.elsevier.com/locate/jde

A local sensitivity analysis for the kinetic Cucker–Smale equation with random inputs

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Received 7 January 2018; revised 29 April 2018

Abstract

We present a local sensitivity analysis for the kinetic Cucker–Smale (C–S) equation with random inputs. This is a companion work to our previous local sensitivity analysis for the particle C–S model. Random inputs in the coefficients of the kinetic C–S equation can be caused by diverse sources such as the incomplete measurement and interactions with unknown environments, and will enter the problem through the communication function or initial data. For the proposed random kinetic C–S equation, we present sufficient conditions for the pathwise well-posedness and flocking estimates. For the local sensitivity analysis, we study the propagation of regularity of the kinetic density function in random space.

MSC: 35Q82; 35Q92; 37H99

Keywords: Cucker–Smale model; Flocking; Local sensitivity analysis; Random communication; Uncertainty quantification

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https://doi.org/10.1016/j.jde.2018.05.013

1. Introduction

The purpose of this paper is to extend the local sensitivity analysis [17] for the particle C–S model to the corresponding mesoscopic model. The jargon "*flocking*" denotes a collective behavior in which particles in a many-body system organize into an ordered motion using the environmental information based on simple rules [4,5,7,41,51,52], e.g., flocking of birds, swarming of fish and herding of sheep, etc. Recently, due to emerging applications [38,44,45] in sensor networks, robot systems and unmanned aerial vehicles, research on the collective dynamics has received lots of attention from diverse scientific disciplines. After Vicsek's seminal work [54] on the collective dynamics, several physical and mathematical models were proposed in literature [11,12,41,42,53]. Among them, our main interest lies on the kinetic C–S equation which can be derived from the mean-field limit from the particle C–S model with random inputs. In literature, the particle and kinetic C–S models have been extensively studied from diverse perspectives, to name a few, collision avoidance [1,9,33], effects of white noises [2,10,19,20,50] and time delay [15], kinetic limit [18,21], dynamics of kinetic model [14,21,22,26,43], uncertainty quantification (UQ) problems [3,6], general networks [8,13,47], variants [34–37] of the C–S model, etc. In this paper we will focus on the kinetic C–S equation with a random communication weight and random initial input.

Let $f = f(t, x, v, z)$ be a one-particle distribution function of the C–S ensemble at position x with velocity $v \in \mathbb{R}^d$, random vector z, here $z = (z_1, \cdots, z_m)$ is a random vector defined on the sample space Ω. The random vector z registers the random effect on the communication weight and initial data. We also assume that each component random variables z_i are i.i.d., and let $\pi = \pi(z)$ be a probability density function for the random vector z. Note that the communication between particles, denoted by $\psi = \psi(x, z)$, and the initial data f^0 are usually determined a priori by empirical data, thus inevitably contain uncertainty, modeled by z. In this situation, the dynamics of the kinetic density f is governed by the mean-field kinetic equation with random inputs:

$$\begin{aligned} &\partial_t f + v \cdot \nabla_x f + \nabla_v \cdot (F_a[f]f) = 0, \quad x, v \in \mathbb{R}^d, \ z \in \Omega, \ t > 0, \\ &F_a[f](t, x, v, z) := -\int_{\mathbb{R}^{2d}} \psi(x - x_*, z)(v - v_*) f(t, x_*, v_*, z) dv_* dx_*, \end{aligned} \tag{1.1}$$

where $\psi(x, z) =: \tilde{\psi}(|x|, z)$ satisfies several structural properties such as the positivity, boundedness, monotonicity and Lipschitz continuity in the first argument: there exists a positive random variable $\psi_M(z) > 0$ such that

$$\begin{aligned} &0 < \psi(x, z) \le \psi_M(z) < \infty, \quad \psi(-x, z) = \psi(x, z), \quad (x, z) \in \mathbb{R}^d \times \Omega, \\ &(\tilde{\psi}(|x_2|, z) - \tilde{\psi}(|x_1|, z))(|x_2| - |x_1|) \le 0, \quad \tilde{\psi}(\cdot, z) \in \mathrm{Lip}(\mathbb{R}; \mathbb{R}_+). \end{aligned} \tag{1.2}$$

For each sample of z, i.e., the randomness is quenched, then (1.1) becomes a deterministic kinetic C–S equation which has been extensively studied in literature [4,7,18,21,22,26,35–37,43].

In this paper, we are mainly interested in the effect of randomness of (1.1) on flocking dynamics and regularity of the solution in the random kinetic equation (1.1), with also random initial data $f^0(x, v, z)$, via the local sensitivity analysis [46,49]. Note that the kinetic density function $f(t, x, v, z + dz)$ can be expanded in z-variable via Taylor's expansion: