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Asymptotic behavior and stability of positive solutions to a spatially heterogeneous predator–prey system ☆

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Abstract

In this paper, we continue to study a spatially heterogeneous predator–prey system where the interaction is governed by a Holling type II functional response, which has been studied in Du and Shi (2007) [14]. We further study the asymptotic profile of positive solutions and give a complete understanding of coexistence region. Moreover, a good understanding of the number, stability and asymptotic behavior of positive solutions is gained for large m. Finally, we further compare the difference of steady-state solutions between $m > 0$ and $m = 0$. It turns out that the spatial heterogeneity of the environment and the Holling type II functional response play a very important role in this model.

MSC: 35J55; 35B30; 35B40; 92D25

Keywords: Spatially heterogeneous; Holling type II response; Asymptotic behavior; Stability

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1. Introduction and main results

In the spatial predator–prey model, the parameters (e.g., the growth rate, the crowding effect, and the predator–prey interaction rates) are traditionally assumed to be constant (see [1,5,9,10,12,13,20–23,26,31] and the references therein). However, due to the heterogeneity of the environment, it is more realistic to assume that the parameters are spatially dependent. Hence a classical predator–prey system with the heterogeneity of the environment and Neumann boundary conditions is the following form:

$$\begin{cases} u_t - d_1\Delta u = \lambda(x)u - a(x)u^2 - b(x)\phi(u)v, & x\in\Omega,\ t>0,\\ v_t - d_2\Delta v = \mu(x)v - d(x)v^2 + c(x)\phi(u)v, & x\in\Omega,\ t>0,\\ \partial_\nu u = \partial_\nu v = 0, & x\in\partial\Omega,\ t>0,\\ u(x,0)=u_0(x)\ge 0,\ v(x,0)=v_0(x)\ge 0, & x\in\overline{\Omega}, \end{cases}$$

where unknown functions $u(x,t)$ and $v(x,t)$ represent the distribution densities of the prey and the predator respectively; the habitat $\Omega\subset\mathbb{R}^n$ is a bounded domain with smooth boundary $\partial\Omega$; the parameter functions are assumed to be nonnegative and smooth in $\overline{\Omega}$ (except $\mu(x)$, which can take negative values); the function $\phi(u)$ represents the functional response of the predator; $\partial_\nu u = \nabla u(x)\cdot\nu(x)$ is the out-flux of u, and $\nu(x)$ is the outer unit normal vector of $\partial\Omega$ at x. Thus the zero-flux boundary condition used in the system implies that it is a closed ecosystem in the habitat Ω.

As known to all, the classical Lotka–Volterra model assumes that $\phi(u)=u$. But when the handling time of each prey is also considered, a more reasonable response function is the Holling type II response $\phi(u)=u/(1+mu)$ for some $m>0$, which was first examined by Holling [18].

To study the effect of the Holling type II response and the degeneracy of the crowding function in the prey equation, Du and Shi [14] considered the following predator–prey system:

$$\begin{cases} u_t - \Delta u = \lambda u - a(x)u^2 - \frac{buv}{1+mu}, & x\in\Omega,\ t>0,\\ v_t - \Delta v = \mu v - v^2 + \frac{cuv}{1+mu}, & x\in\Omega,\ t>0,\\ \partial_\nu u = \partial_\nu v = 0, & x\in\partial\Omega,\ t>0,\\ u(x,0)=u_0(x)\ge 0,\ v(x,0)=v_0(x)\ge 0, & x\in\overline{\Omega}, \end{cases}$$

and the corresponding steady-state problem is

$$\begin{cases} -\Delta u = \lambda u - a(x)u^2 - \frac{buv}{1+mu}, & x\in\Omega,\\ -\Delta v = \mu v - v^2 + \frac{cuv}{1+mu}, & x\in\Omega,\\ \partial_\nu u = \partial_\nu v = 0, & x\in\partial\Omega, \end{cases} \tag{1.1}$$

where λ, μ, b, c, m are constants, all positive except μ, which may take negative values; $a(x)$ is a nonnegative smooth function on $\overline{\Omega}$ satisfying

$$a(x)\equiv 0,\quad x\in\overline{\Omega}_0 \ \text{ and } \ a(x)>0,\quad x\in\overline{\Omega}\setminus\overline{\Omega}_0,$$

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