Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations 265 (2018) 2921–2967

Journal of Differential Equations

www.elsevier.com/locate/jde

Explicit approximations for nonlinear switching diffusion systems in finite and infinite horizons ☆

Hongfu Yang, Xiaoyue Li *

School of Mathematics and Statistics, Northeast Normal University, Changchun, Jilin, 130024, PR China

Received 7 November 2017; revised 21 February 2018
Available online 4 May 2018

Abstract

Focusing on hybrid diffusion dynamics involving continuous dynamics as well as discrete events, this article investigates the explicit approximations for nonlinear switching diffusion systems modulated by a Markov chain. Different kinds of easily implementable explicit schemes have been proposed to approximate the dynamical behaviors of switching diffusion systems with locally Lipschitz continuous drift and diffusion coefficients in both finite and infinite intervals. Without additional restriction conditions except those which guarantee the exact solutions possess their dynamical properties, the numerical solutions converge strongly to the exact solutions in finite horizon, moreover, realize the approximation of long-time dynamical properties including the moment boundedness, stability and ergodicity. Some simulations and examples are provided to support the theoretical results and demonstrate the validity of the approach.

MSC: 65H10; 60J60; 60J27; 60J22

Keywords: Explicit scheme; Switching diffusion systems; Local Lipschitz condition; Strong convergence; Stability; Invariant measure

☆ This work was supported by National Natural Science Foundation of China (11171056, 11471071, 11671072), the Natural Science Foundation of Jilin Province (No. 20170101044JC), the Education Department of Jilin Province (No. JJKH20170904KJ).

* Corresponding author.
E-mail address: lixy209@nenu.edu.cn (X. Li).

https://doi.org/10.1016/j.jde.2018.04.052

1. Introduction

The switching diffusion systems (SDSs) modulated by Markov chains involving continuous dynamics and discrete events, have drawn more and more attention to many researches. Much of the study originated from applications arising from biological systems, financial engineering, manufacturing systems, wireless communications (see, e.g., [1–5] and the references therein). Compared with those of the subsystems the dynamics of SDSs are seemingly much different. For instance, considering a predator–prey ecosystem switching between two environments randomly, Takeuchi et al. in [4] revealed that both subsystems develop periodically but switching between them makes them neither permanent nor dissipative. Pinsky and his coauthors in [6,7] provided several interesting examples to show that the switching system is recurrent (resp. transient) even if its subsystems are transient (resp. recurrent). Due to the coexistence of continuous dynamics and discrete events, the dynamics of SDSs are full of uncertainty and challenge.

Since solving SDSs is almost unavailable, numerical schemes or approximation techniques become viable alternatives. The explicit Euler–Maruyama (EM) scheme is popular for approximating diffusion systems and SDSs with global Lipschitz coefficients [2,5,8]. However, the coefficients of many important diffusion systems and SDSs are only locally Lipschitz and superlinear (see, e.g., [3,5,9,10] and the references therein). Hutzenthaler et al. [10, Theorem 2.1] showed that the absolute moments of the EM approximation for a large class of diffusion systems with superlinear growth coefficients diverge to infinity at a finite time point $T \in (0, \infty)$. The implicit EM scheme is better than the explicit EM scheme in that its numerical solutions converge strongly to the exact solutions of diffusion and switching diffusion systems with the one-sided Lipschitz drift coefficient and global Lipschitz diffusion coefficient (see Higham et al. [11], Mao and Yuan [2, pp. 134–153]). Nevertheless, additional computational efforts are required for its implementation since the solution of an algebraic equation has to be found before each iteration. Due to the advantages of explicit schemes (e.g., simple structure and cheap computational cost), a few modified EM methods have been developed for diffusion systems with nonlinear coefficients including the tamed EM method [9,12–14], the tamed Milstein method [15], the stopped EM method [16] and the truncated EM method [17]. These modified EM methods have shown their abilities to approximate the solutions of nonlinear diffusion systems. However, to the best of our knowledge these methods are not developed, even unavailable for a large class of nonlinear SDSs. For instance, Hamilton [18] remarked that the economy may either be in a fast growth or slow growth phase with the regime switching governed by the outcome of a Markov chain. Consider a two-dimensional nonlinear stochastic volatility model switching randomly between a fast growth phase

$$\mathrm{d}X(t) = 2.5X(t)\Big(1 - |X(t)|\Big)\mathrm{d}t + \begin{pmatrix} -1 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix} |X(t)|^{3/2}\mathrm{d}B(t), \tag{1.1}$$

and a slow growth phase

$$\mathrm{d}X(t) = \Big((1,2)^T - X(t)\Big)\mathrm{d}t + \begin{pmatrix} 0.2 & -0.5 \\ 1 & 0.4 \end{pmatrix} |X(t)|\mathrm{d}B(t), \tag{1.2}$$

modulated by a Markov chain $r(t)$. This SDS is one of popular volatility models used for pricing option in Finance (see, e.g., [1,3,18,19] and the references therein), especially, for pricing VIX options. Based on its importance and no closed-form, one of our aims is to construct the explicit