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Construction of type II blowup solutions for the 1-corotational energy supercritical wave maps

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Abstract

We consider the energy supercritical wave maps from $\mathbb{R}^d$ into the d-sphere $\mathbb{S}^d$ with $d \geq 7$. Under an additional assumption of 1-corotational symmetry, the problem reduces to the one dimensional semilinear wave equation

$$\partial_t^2 u = \partial_r^2 u + \frac{(d-1)}{r}\partial_r u - \frac{(d-1)}{2r^2}\sin(2u).$$

We construct for this equation a family of $\mathcal{C}^\infty$ solutions which blow up in finite time via concentration of the universal profile

$$u(r,t) \sim Q\left(\frac{r}{\lambda(t)}\right),$$

where Q is the stationary solution of the equation and the speed is given by the quantized rates

$$\lambda(t) \sim c_u(T-t)^{\frac{\ell}{\gamma}}, \quad \ell \in \mathbb{N}^*, \ \ell > \gamma = \gamma(d) \in (1,2].$$

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The construction relies on two arguments: the reduction of the problem to a finite-dimensional one thanks to a robust universal energy method and modulation techniques developed by Merle, Raphaël and Rodnianski [49] for the energy supercritical nonlinear Schrödinger equation, then we proceed by contradiction to solve the finite-dimensional problem and conclude using the Brouwer fixed point theorem.
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1. Introduction

Let (N, h) be a complete smooth Riemannian manifold of dimension d with $\partial N = \emptyset$. We denote spacetime coordinates on $\mathbb{R}^{1+d}$ as $(t, x) = (x_\alpha)$ with $0 \le \alpha \le d$. A wave map $\Phi : \mathbb{R}^{1+d} \mapsto N$ is formally defined as a critical point of the Lagrangian

$$\mathcal{L}(\Phi, \partial\Phi) = \int_{\mathbb{R}^{1+d}} g^{\alpha\mu}\Big\langle \partial_\alpha \Phi, \partial_\mu \Phi \Big\rangle_h dtdx,$$

where $g = \mathrm{diag}(-1, 1, \cdots, 1)$ is the Minkowski metric on $\mathbb{R}^{1+d}$ and $\partial_\alpha = \frac{\partial}{\partial x_\alpha}$. In the local coordinates on (N, h), the critical points of $\mathcal{L}$ satisfy the equation

$$\Box_g \Phi^k + g^{\alpha\mu}\Gamma^k_{ij}(\Phi)\partial_\alpha \Phi^i \partial_\mu \Phi^j = 0, \quad 1 \le k \le d, \tag{1.1}$$

where Γ^k_{ij} are Christoffel symbols associated to the metric h of the target manifold N, and $\Box_g$ stands for the Laplace–Beltrami operator on $(\mathbb{R}^{1+d}, g)$ defined by

$$\Box_g u = \partial_{tt} - \Delta.$$

A special case is when the target manifold $N = \mathbb{S}^d \hookrightarrow \mathbb{R}^{1+d}$, equation (1.1) becomes

$$\partial_t^2 \Phi - \Delta\Phi = \Phi(|\nabla\Phi|^2 - |\partial_t \Phi|^2). \tag{1.2}$$

Under the assumption of 1-corotational symmetry, namely that the solution takes the form

$$\Phi(x, t) = \begin{pmatrix} \cos(u(|x|, t)) \\ \frac{x}{|x|}\sin(u(|x|, t)) \end{pmatrix},$$

equation (1.2) reduces to the semilinear wave equation

$$\begin{cases} \partial_t^2 u = \partial_r^2 u + \frac{(d-1)}{r}\partial_r u - \frac{(d-1)}{2r^2}\sin(2u), \\ (u, \partial_t u)\,|_{t=0} = (u_0, u_1), \end{cases} \tag{1.3}$$

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