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Global existence and large time behavior for a two-dimensional chemotaxis–shallow water system

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Abstract

This paper concerns the chemotaxis–shallow water system in two dimensional whole spaces. We first establish the global existence of strong solution under the condition that the initial data are close to the constant equilibrium state in $H^N(\mathbb{R}^2)(N \geq 3)$-framework. Then, the optimal time decay rates of the global solution are obtained by the method of energy estimates, if the initial data belong to $\dot{H}^{-s}(\mathbb{R}^2)(0 < s < 1)$ additionally.

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Keywords: Chemotaxis; Shallow water system; Global existence; Decay rates

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1. Introduction

This paper is concerned with the following chemotaxis–shallow water system

$$\begin{cases} n_t + \operatorname{div}(nu) = D_n \Delta n - \nabla \cdot (n\chi(c)\nabla c), \\ c_t + \operatorname{div}(cu) = D_c \Delta c - nf(c), \\ h_t + \operatorname{div}(hu) = 0, \\ hu_t + hu \cdot \nabla u + h^2 \nabla n + \dfrac{1}{2}(1+n)\nabla h^2 = \mu \Delta u + (\mu + \lambda)\nabla(\operatorname{div} u), \end{cases} \tag{1.1}$$

which was proposed in [5] to describe the dynamics of the oxygen and aerobic bacteria in the incompressible fluids with free surface. Here n, c represent the bacterial density, the chemoattractant concentration respectively. h is the fluid height and $u \in \mathbb{R}^2$ is the fluid velocity field. The constants D_n and D_c are the corresponding diffusion coefficients for the cells and sub-strate. The chemotactic sensitivity $\chi(c)$ and the consumption rate of the substrate by the cells $f(c)$ are supposed to be given smooth functions. The constants μ and λ are the shear viscosity and the bulk viscosity coefficients respectively with the following physical restrictions:

$$\mu > 0, \quad \mu + \lambda \geq 0.$$

In order for the system (1.1) to be well-posed, it should be supplemented with some initial conditions

$$(n, c, h, u)(x, t)|_{t=0} = (n_0(x), c_0(x), h_0(x), u_0(x)). \tag{1.2}$$

As the space variable tends to infinity, we assume

$$\lim_{|x| \to \infty} (n_0 - \bar{n}, c_0, h_0 - 1, u_0)(x) = 0, \tag{1.3}$$

where $\bar{n}$ is a positive constant. For the whole space, the boundary condition is hidden in the decay of solutions at spatial infinity.

Chemotaxis is a biological process in which cells move toward a chemically more favorable environment. The consequence of chemotaxis is that the cells changes its movement toward (or away from) a higher concentration of the chemical stimulus. In order to model this common phenomenon in biology, Keller and Segel [17,18] first derived a chemotaxis model. Since then, a lots of modified chemotaxis models appeared, such as aggregative patterns of bacteria [21,22], slime mold formation [15], fish pigmentation patterning [23], angiogenesis in tumor progression [4], primitive streak formation [24], wound healing [25], and so on.

In fact, bacteria or microorganisms often live in fluid. Both the oxygen concentration and bacteria density are transported by the fluid and diffuse through the fluid. Therefore, the biology of aerotaxis is intimately related to the surrounding physics. Tuval et al. [30] proposed a coupled system of the chemotaxis model and the viscous incompressible fluid. The model is as follows