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Global existence of weak solutions to two dimensional compressible viscoelastic flows

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Abstract

The global existence of weak solutions of the compressible viscoelastic flows in two spatial dimensions is studied in this paper. We show the global existence if the initial velocity $\mathbf{u}_0$ is small in H^η with an arbitrary $\eta \in (0, \frac{1}{2})$ and the perturbation of $(\rho_0, \mathtt{F}_0)$ around the constant state $(1, I)$ are small in $L^2 \cap \dot{B}^{\frac{2}{p}}_{p,1}$ with $p \in (\frac{-1+\sqrt{1+16\eta}}{2\eta}, 4)$. One of the main ingredients is that the velocity and the "effective viscous flux" $\mathcal{G}_i$ are sufficiently regular for positive time. The regularity of $\mathcal{G}_i$ helps to obtain the L^∞ estimate of density and deformation gradient, and hence eliminates the possible concentration and oscillation issues.

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Keywords: Compressible viscoelastic fluid; Weak solution; Effective viscous flux; Global well-posedness

1. Introduction

The flow of compressible viscoelastic fluids can be described by the following equations which are equivalent to the classical Oldroyd-B model (see [8,20,23,24]):

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$$\begin{cases} \partial_t \rho + \operatorname{div}(\rho \mathbf{u}) = 0, \\ \partial_t(\rho \mathbf{u}) + \operatorname{div}(\rho \mathbf{u} \otimes \mathbf{u}) - \mu \Delta \mathbf{u} - \xi \nabla \operatorname{div} \mathbf{u} + \nabla P(\rho) = \operatorname{div}(\rho \mathtt{F}\mathtt{F}^\top), \\ \partial_t \mathtt{F} + \mathbf{u} \cdot \nabla \mathtt{F} = \nabla \mathbf{u} \mathtt{F}, \end{cases} \tag{1.1}$$

where $\rho \in \mathbb{R}$ is the density, $\mathbf{u} \in \mathbb{R}^2$ denotes the velocity of the fluid, $\mathtt{F} \in \mathcal{M}$ is the deformation gradient ($\mathcal{M}$ is the set of 2×2 matrices with $\rho \det \mathtt{F} = 1$), and $P(\rho)$ is the pressure of the fluid, which is assumed to take the isentropic law $P(\rho) = \rho^\gamma$. The viscosities μ and ξ satisfies

$$\mu > 0 \quad \text{and} \quad \xi \geq 0.$$

The system (1.1) is supplemented by the initial data

$$(\rho, \mathbf{u}, \mathtt{F})|_{t=0} = (\rho_0, \mathbf{u}_0, \mathtt{F}_0)(x) \quad \text{for all} \quad x \in \mathbb{R}^2. \tag{1.2}$$

Whenever solutions $(\rho, \mathbf{u}, \mathtt{F})$ to (1.1) are smooth, it was well-known facts, see [6,20,23], that

$$\begin{cases} \operatorname{div}(\rho \mathtt{F}^\top)(t) = 0, \\ \mathtt{F}_{lk} \partial_{x_l} \mathtt{F}_{ij}(t) = \mathtt{F}_{lj} \partial_{x_l} \mathtt{F}_{ik}(t), \\ \rho \det \mathtt{F}(t) = 1 \end{cases} \tag{1.3}$$

for all $t > 0$ whenever (1.3) holds initially.

In the incompressible regime, for classical solutions of (1.1)–(1.2) or related Oldroyd-B models, authors in [6,8,20,23,24] have established the global existence and well-posedness of solutions to (1.1)–(1.2) in various functional spaces whenever the initial data is a small perturbation around the equilibrium $(1, 0, I)$, where I is the identity matrix. We refer to readers also [2,9,16,26,27,30–32] and references therein for local and global existence of solutions of closely related models and [33] for numerical evidences of singularities. In particular in [21,22,30,31], the authors use the hyperbolic nature of the system (1.1)–(1.2) when μ and ξ vanish to establish an interesting global existence result for classical solution in a subspace of H^s ($s \geq 8$) when the initial date is also a small perturbation in that space of $(1, 0, I)$ due to the dispersive structure. The global existence of weak solutions for the incompressible version of (1.1) with small perturbations near the equilibrium is considered in [16]. The regularity in terms of bounds on the elastic stress tensor was established in [7,19]. In [8] authors proved global existence for small data with large gradients for Oldroyd-B. Regularity for diffusive Oldroyd-B equations for large data were obtained in the creeping flow regime in [2,9]. For the Oldroyd-B type models with a finite relaxation time the global existence of weak solutions with natural initial data had been verified in [4,26] under the corotational assumption. A remarkable global existence result for weak solutions for the FENE dumbbell model with suitable initial data has been constructed by Masmoudi in [27,28] through a detailed analysis of the defect measure associated with the approximations. In the compressible regime, the wellposedness and optimal decay of classical solutions of (1.1) have been established in [14,15,29] and the wellposedness of (1.1) as viscosities vanish has been studied in [17] when the electrical field is present. A global existence with large initial data is studied in [2,3] for the incompressible Oldroyd-B model with a nonlinear diffusion term.

The construction of global solutions in [6,14,20,22–24,29] depends crucially on various conserved quantity (1.3) (see also [8]). Unfortunately, when the regularity of the initial data is weak,