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Asymptotic behavior of Boussinesq system of KdV–KdV type

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Abstract

This work deals with the local rapid exponential stabilization for a Boussinesq system of KdV–KdV type introduced by J. Bona, M. Chen and J.-C. Saut. This is a model for the motion of small amplitude long waves on the surface of an ideal fluid. Here, we will consider the Boussinesq system of KdV–KdV type posed on a finite domain, with homogeneous Dirichlet–Neumann boundary controls acting at the right end point of the interval. Our goal is to build suitable integral transformations to get a feedback control law that leads to the stabilization of the system. More precisely, we will prove that the solution of the closed-loop system decays exponentially to zero in the $L^2(0, L)$-norm and the decay rate can be tuned to be as large as desired if the initial data is small enough.

MSC: primary 93B05, 93D15, 35Q53

Keywords: Boussinesq system KdV–KdV type; Stabilization; Feedback integral transformation; Backstepping method

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1. Introduction

1.1. Setting of the problem

The classical Boussinesq systems were first derived by Boussinesq in [4], to describe the two-way propagation of small amplitude, long wave length gravity waves on the surface of water in a canal. These systems and their higher-order generalizations also arise when modeling the propagation of long-crested waves on large lakes or on the ocean and in other contexts. Recently, in [2], the authors derived a four-parameter family of Boussinesq systems to describe the motion of small amplitude long waves on the surface of an ideal fluid under the gravity force and in situations where the motion is sensibly two dimensional. More precisely, they studied a family of systems of the form

$$\begin{cases}\eta_t + w_x + (\eta w)_x + a w_{xxx} - b\eta_{xxt} = 0,\\ w_t + \eta_x + w w_x + c\eta_{xxx} - d w_{xxt} = 0.\end{cases} \tag{1.1}$$

In (1.1), η is the elevation from the equilibrium position, and $w = w_\theta$ is the horizontal velocity in the flow at height θh, where h is the undisturbed depth of the liquid. The parameters a, b, c, d, that one might choose in a given modeling situation, are required to fulfill the relations

$$a + b = \frac{1}{2}\left(\theta^2 - \frac{1}{3}\right), \qquad c + d = \frac{1}{2}(1-\theta^2) \geq 0, \qquad \theta \in [0,1], \tag{1.2}$$

where $\theta \in [0,1]$ specifies which horizontal velocity the variable w represents (cf. [2]). Consequently,

$$a + b + c + d = \frac{1}{3}.$$

As it has been proved in [2], the initial value problem for the linear system associated with (1.1) is well-posed on $\mathbb{R}$ if either C_1 or C_2 is satisfied, where

$$\begin{aligned} &(C_1) \quad b, d \geq 0,\ a \leq 0,\ c \leq 0;\\ &(C_2) \quad b, d \geq 0,\ a = c > 0. \end{aligned}$$

When $b = d = 0$ and (C_2) is satisfied, then necessarily $a = c = 1/6$. Nevertheless, the scaling $x \to x/\sqrt{6}$, $t \to t/\sqrt{6}$ gives an system equivalent to (1.1) for which $a = c = 1$, namely

$$\begin{cases}\eta_t + w_x + w_{xxx} + (\eta w)_x = 0, & \text{in } (0,L) \times (0,+\infty),\\ w_t + \eta_x + \eta_{xxx} + w w_x = 0, & \text{in } (0,L) \times (0,+\infty),\\ \eta(x,0) = \eta_0(x), \quad w(x,0) = w_0(x), & \text{in } (0,L),\end{cases} \tag{1.3}$$

which is the so-called *Boussinesq system of Korteweg–de Vries–Korteweg–de Vries type*.

Therefore, the interest of this work is to give a positive answer for the following stabilization problem: