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Nonlinear stability of strong traveling waves for the singular Keller–Segel system with large perturbations

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Abstract

This paper is concerned with the nonlinear stability of traveling wave solutions for a conserved system of parabolic equations derived from a singular chemotaxis model describing the initiation of tumor angiogenesis. When the initial datum is a continuous small perturbation with zero integral from the spatially shifted traveling wave, the asymptotic stability of the large-amplitude (strong) traveling waves has been established in a series of works [29,34,35] by the second author with his collaborators. In this paper, we shall show that similar stability results indeed hold true for large and discontinuous initial data (i.e. the initial perturbation from the traveling wave could be discontinuous and has large oscillations) such as Riemann data with large jumps. To the best of our knowledge, this paper provides a first result on the asymptotic stability of large-amplitude traveling waves with large initial perturbation for a system of conservation laws, although similar results have been available for the scalar equations (cf. [8,42]). We also extend existing results to the initial data with lower regularity.

MSC: 35A01; 35B40; 35B44; 35K57; 35Q92; 92C17

Keywords: Chemotaxis; Traveling wave solutions; Nonlinear stability; Logarithmic sensitivity; Large perturbation; Discontinuous data

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1. Introduction

It is well known that chemotaxis, the movement of organism towards higher concentration of chemical substance, can produce rich wave patterns in different circumstances, such as traveling band of bacterial toward the oxygen [2], the outward propagation of concentric ring waves by *E. coli* [4], the spiral wave patterns during the aggregation of *Dictyostelium discoideum* [9] and the migration of *Myxococcus xanthus* in the early stage of starvation-induced fruiting body development [55]. The mathematical study of chemotactic traveling waves was started by Keller and Segel in their seminal paper [22] wherein the following model

$$\begin{cases} u_t = [Du_x - \chi u(\ln c)_x]_x, \\ c_t = \varepsilon c_{xx} - uc^m \end{cases} \tag{1.1}$$

was proposed to describe the propagation of traveling bands of chemotactic bacteria observed in the celebrated experiment of Adler [2], where $u(x,t)$ denotes the bacterial density and $c(x,t)$ the oxygen concentration. $D > 0$ and $\varepsilon \geq 0$ are bacterial and chemical diffusion coefficients, respectively, $\chi > 0$ is the chemotactic coefficient and $m \geq 0$ is the oxygen consumption rate.

When $0 \leq m < 1$, Keller and Segel [22] managed to use the model (1.1) with $\varepsilon = 0$ to interpret the traveling bands observed in the experiment of [2], followed with a serious of works for the case $\varepsilon \geq 0$ (cf. [21,37,41,44,47]). When $m > 1$, the model (1.1) does not admit traveling wave solutions (e.g., see [47,53]). In the borderline case $m = 1$, the model (1.1) with $\varepsilon > 0$ was first used by Rosen [45,46] to describe the chemotactic movement of motile aerobic bacterial toward oxygen, and later was employed to describe the directed movement of endothelial cells toward the signaling molecule vascular endothelial growth factor (VEGF) during the initiation of angiogenesis (cf. [6,7,24,25]).

While the existence of traveling wave solutions of the Keller–Segel model (1.1) with $\varepsilon \geq 0$ and $m \geq 0$ has been well established (see a review paper [53]), the stability of traveling wave solutions still remains as a very challenging question due to the singularity caused by the logarithmic sensitivity $\ln c$ whose mathematical derivation and biological relevance have been later presented in [20,43]. For the case $0 \leq m < 1$, expect some instability result [41] and classification of essential spectrum (cf. [38,39]) based on spectral analysis, no stability results on traveling wave solutions are available so far. However, in the case $m = 1$, the stability of traveling wave solutions to (1.1) with small $\varepsilon > 0$ (or $\varepsilon = 0$) has been gradually obtained (cf. [26,29,31–35]) by the (weighted) energy estimates. The success of these results heavily reply on the following Cole–Hope type transformation (cf. [23,34])

$$v = -(\ln c)_x = -\frac{c_x}{c},$$

which converts (1.1) with $m = 1$ into a parabolic system of conservation laws without singularity

$$\begin{cases} u_t - \chi(uv)_x = Du_{xx}, \\ v_t + (\varepsilon v^2 - u)_x = \varepsilon v_{xx}. \end{cases} \tag{1.2}$$

The transformation (1.2) significantly clears the obstruction caused by the logarithmic singularity in the original Keller–Segel system (1.1). Consequently a great deal of interesting results have been carried out for the transformed system (1.2) from various perspectives. For the global dynamics of classical solutions and nonlinear stability of traveling wave solutions of (1.2), we refer

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