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ScienceDirect

J. Differential Equations 265 (2018) 1839–1874

Journal of Differential Equations

www.elsevier.com/locate/jde

On a new model for fatigue and phase transition in an oscillating elastoplastic plate ☆

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Received 10 January 2017; revised 2 February 2018
Available online 17 April 2018

Abstract

We consider a thermodynamic model for fatigue accumulation in an oscillating elastoplastic plate based on the two hypotheses that the fatigue accumulation rate is proportional to the plastic part of the dissipation rate and that the material can partially recover by the effect of melting. For the full model, consisting of the momentum and energy balances, an evolution equation for the fatigue rate and a differential inclusion for the phase dynamics, we prove existence of a solution in the given time interval.

☆ Supported by GAČR Grant 15-12227S, and by the institutional support for the development of research organizations IČ 47813059. The work of Michela Eleuteri is supported by GNAMPA (Gruppo Nazionale per l'Analisi Matematica, la Probabilità e le loro Applicazioni) of INdAM (Istituto Nazionale di Alta Matematica), through the projects GNAMPA 2016 "Regolarità e comportamento asintotico di soluzioni di equazioni paraboliche" (coord. Prof. S. Polidoro) and GNAMPA 2017 "Regolarità per problemi variazionali d'ostacolo e liberi" (coord. Prof. M. Focardi), and by the University of Modena and Reggio Emilia through the project FAR2015 "Equazioni differenziali: problemi evolutivi, variazionali ed applicazioni" (coord. Prof. S. Polidoro) and FAR2017 "Equazioni differenziali: problemi evolutivi, variazionali ed applicazioni" (coord. Prof. S. Gatti).

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https://doi.org/10.1016/j.jde.2018.04.020

1. Introduction

The present paper constitutes another step of a project that the authors started in 2012 in cooperation with Pavel Krejčí. The aim of the project has been to propose a new model for the cyclic fatigue accumulation in transversally oscillating elastoplastic structures (beams and plates) and to study its properties. The starting point has been the work done in [21] (see also [23]). In this paper, the Kirchhoff-Love method of reducing the 3D problem of transversal oscillations of a solid elastoplastic beam with the single yield von Mises plasticity law has been used to obtain the beam equation with a multiyield hysteresis Prandtl–Ishlinskii constitutive operator. Starting from these ideas, we derived in [6] our new model and showed its thermodynamical consistency. From the modeling point of view, the main novelty has been the use of the Prandtl–Ishlinskii formalism to discuss the process of cyclic fatigue accumulation, where, accordingly with the observation in [5], we assumed a *proportionality between accumulated fatigue and dissipated energy*. This viewpoint is new in the literature; on one hand because we focus on the dynamics of the processes (compared, for instance, with [13], [29], [30] that go into the direction of rate-independent damage processes in nonlinear elasticity) and on the other hand because the approach used in other papers that interpret damage processes as a kind of phase transition in the material (see for instance [1], [2], [3], [28]) is based on the idea that damage processes are driven by large deformations.

The mathematical treatment of our new model started from the paper [7], where we dealt with the case of an oscillating elastoplastic plate assuming that both the elastic and the plastic characteristics change with increasing fatigue, under the simplified situation of given temperature history. In this case we obtained existence and uniqueness of a solution locally in time; accordingly to the model indeed, material softening takes place under increasing fatigue and material failure is manifested in finite time.

From [8], we started dealing with the non-isothermal case. The full system of equations consisted of the momentum balance equation coupled with the energy balance and the fatigue accumulation equation. In [8], where we treated the 1D case (beam), we showed existence and uniqueness of a strong solution in a time interval depending on the size of the data, in the simplified setting where only the elastic component of the model depends on the fatigue parameter. The extension to the 2D case (plate) has been considered in [9], where we proved existence of solutions for the whole time interval, taking a different perspective (usually considered in engineering analysis), i.e. assuming that the fatigue accumulation rate is proportional only to the plastic part of the dissipation rate. From the mathematical viewpoint, the problem does not exhibit singularities anymore and the expected solutions turn to be global in time.

Fatigue accumulation in a moving elastoplastic material in contact with an elastoplastic obstacle was studied in [11]. We used a new approach, suggested in the recent papers [17], [18], where the problem for the unknown displacement and temperature is formulated using hysteresis operators as solution operators of the underlying variational inequalities to model the contact boundary conditions. The plastic deformation of elastoplastic bodies in contact dissipates energy which is transformed into heat. This in turn increases the temperature of the body and, by thermal expansion, the motion of the body is affected. The existence result for this problem, consisting of the momentum and energy balance equations and an evolution equation for the fatigue is obtained, using a priori estimates established for the space discretized problem. The uniqueness result follows from the Lipschitz continuity of the nonlinearities.

From [15], [4] we pursued the study of fatigue accumulation in oscillating elastoplastic structures by presenting a new phase field model under the additional hypothesis that the material can