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Global solution to initial boundary value problem for gas dynamics in thermal nonequilibrium

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Abstract

The global existence and long time behavior of smooth solutions to an initial boundary value problem for the system of thermal nonequilibrium gas dynamics are studied in this paper.

Keywords: Global solutions; Boundary conditions; Thermal nonequilibrium; Weak dissipation

1. Introduction

The motion of a gas in local thermodynamic equilibrium is governed by the compressible Euler equations. In Lagrangian coordinates, the equations for one dimensional flow read (cf. [3]):

$$\begin{cases} v_t - u_x = 0, \\ u_t + p_x = 0, \\ (e + \frac{u^2}{2})_t + (pu)_x = 0, \end{cases} \tag{1.1}$$

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where v, u, p and e are, respectively, the specific volume, velocity, pressure and internal energy of the gas. During rapid changes in the flow the internal energy e may lag behind the equilibrium value corresponding to the ambient pressure and density. The translational energy adjusts quickly, but the rotational and vibrational energy may take an order of magnitude longer. If we suppose that α of the degrees of freedom adjust instantaneously but a further α_f degrees of freedom take longer to relax, we may take (cf. [13,14]):

$$e = \frac{\alpha}{2} pv + q =: e_1 + q, \tag{1.2}$$

where q is the energy in the lagging degrees of freedom. In equilibrium, q would have the value

$$q_{\text{equil}} = \frac{\alpha_f}{2} pv. \tag{1.3}$$

A simple overall equation to represent the relaxation is (in Lagrangian coordinates):

$$q_t = -\frac{1}{\tau}(q - \frac{\alpha_f}{2} pv), \tag{1.4}$$

where $\tau > 0$ is the relaxation time. Therefore, in thermal nonequilibrium, the following system of equations is used to model the gas motion:

$$\begin{cases} v_t - u_x = 0, \\ u_t + p_x = 0, \\ (\frac{\alpha}{2} pv + q + \frac{u^2}{2})_t + (pu)_x = 0, \\ q_t = -\frac{1}{\tau}(q - \frac{\alpha_f}{2} pv). \end{cases} \tag{1.5}$$

If the relaxation time τ is taken to be so short that $q = \frac{\alpha_f}{2} pv$ is an adequate approximation to the last equation in (1.5), we have the following equilibrium theory:

$$\begin{cases} v_t - u_x = 0, \\ u_t + p_x = 0, \\ (\frac{\alpha+\alpha_f}{2} pv + \frac{u^2}{2})_t + (pu)_x = 0. \end{cases} \tag{1.6}$$

System (1.5) is a hyperbolic system with relaxation, and the relaxation term may induce certain dissipative effects which is important to the decay and global regularity of solutions. It is well-known that singularities can form for genuinely nonlinear hyperbolic systems without dissipation ([7,6]), no matter how smooth and small the initial data are. However, for small perturbations, the dissipative effects may smooth out singularities and lead to the global existence of smooth solutions. Indeed, for some inhomogeneous quasilinear hyperbolic systems, global existence and large time behavior have been extensively studied ([1,2,4,5,10–12,16–19]), due to the time decay of solutions caused by the dissipative effects induced by the inhomogeneous terms through the strong coupling with the flux functions of which a typical version is the Shizuta–Kawashima condition, introduced for hyperbolic–parabolic systems originally, [15]. However, as pointed out in [21], system (1.5) of gas dynamics in thermal non-equilibrium does not satisfy those conditions

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