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Positive solutions for a Kirchhoff problem with vanishing nonlocal term [☆]

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Abstract

In this paper we study the Kirchhoff problem

$$\begin{cases} -m(\|u\|^2)\Delta u = f(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

in a bounded domain, allowing the function m to vanish in many different points. Under an appropriated *area condition*, by using a priori estimates, truncation techniques and variational methods, we prove a multiplicity result of positive solutions which are ordered in the $H_0^1(\Omega)$-norm.

MSC: 35J20; 35J25; 35Q74

Keywords: Kirchhoff type equation; Degenerate coefficient; Variational method

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1. Introduction

Let $\Omega \subset \mathbb{R}^N$, $N \geq 1$ be a smooth bounded domain. We are interested in this paper to study the existence of positive solutions for the problem

$$\begin{cases} -m(\|u\|^2)\Delta u = f(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{P}$$

where $\|u\| := |\nabla u|_2$ is the usual norm in the Sobolev space $H_0^1(\Omega)$ and $m : [0, \infty) \to \mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ are suitable continuous functions. Along the paper, $|\cdot|_p$ will denote the $L^p(\Omega)$-norm.

Problem (P) is the N-dimensional version, in the stationary case, of the *Kirchhoff equation* introduced in [8]. Over the past years several authors have undertaken reasonable efforts to investigate stationary Kirchhoff problems like (P), by considering different general assumptions on functions m and f. Without any intention to provide a survey about the subject, we would like to refer the reader to the papers [1,4,5,7,9,10] and the references therein.

Beside the importance of their contributions, all the previous mentioned papers require the function m to be bounded from below by a positive constant. In this way the problem (P) is not degenerate and many approaches involving variational and topological methods can be used in a straightforward and effective way in order to get solutions.

On the other hand, in the recent paper [2] the positivity assumption on m is relaxed. Indeed Ambrosetti and Arcoya consider the degenerate coefficient m by allowing $m(0) = 0$ and/or $\lim_{t\to+\infty} m(t) = 0$. However in such a paper the condition $m(t) > 0$ for $t > 0$ is maintained. See [2, Section 3].

Motivated by the above facts and by papers [3,6], where multiplicity results are obtained for an elliptic problem under a local nonlinearity which vanishes in different points, a natural question concerns the existence of many solutions for problem (P) in the case of a degenerate m, that is, when it can vanish in many different points. Indeed this is the object of this paper to which we give a positive answer. Roughly speaking, if m vanishes in K distinct points and a suitable area condition is imposed, then the problem has K positive and ordered solutions.

To be more precise, let us start to give the assumptions on the problem. We require the following conditions on the functions m and f:

(m) there exist positive numbers $0 < t_1 < t_2 < \ldots < t_K$ such that
- $m(t_k) = 0$ for all $k \in \{1, \ldots, K\}$,
- $m > 0$ in (t_{k-1}, t_k), for all $k \in \{1, \ldots, K\}$; we agreed that $t_0 = 0$,

(f) there exists $s_* > 0$ such that $f(t) > 0$ in $(0, s_*)$ and $f(s_*) = 0$.

We define the following truncation of the function f:

$$f_*(t) = \begin{cases} f(0) & \text{if } t < 0, \\ f(t) & \text{if } 0 \leq t < s_*, \\ 0 & \text{if } s_* \leq t, \end{cases} \tag{1.1}$$

which is of course continuous, and let $F_*(t) = \int_0^t f_*(s)ds$.

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