ARTICLE IN PRESS YJDEQ:9293

Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Quasi-periodic self-excited travelling waves for damped beam equations

Nemanja Kosovalić

University of South Alabama, United States

Received 15 July 2017; revised 10 April 2018

Abstract

A two dimensional beam equation is considered. It is shown that for a dense set of model parameters, the presence of linear and nonlinear damping terms of opposite sign, produce small amplitude quasi-periodic travelling waves, which are continuations of two branches of rotating waves. Lyapunov–Schmidt reduction and only the classical implicit function theorem are used.

Keywords: Beam equation; Codimension two bifurcation; Symmetry; Quasi-periodic; Travelling wave

1. Introduction

Consider the following two-dimensional damped beam equation:

$$\begin{cases} \partial_t^2 u(t,x) + \gamma \Delta^2 u(t,x) + \alpha \partial_t u(t,x) + \beta \partial_t \Delta^2 u(t,x) + m u(t,x) = (\partial_t u(t,x))^p \\ x \in \mathbf{T}^2. \end{cases} \tag{1}$$

Here $\mathbf{T}^2 := \mathbf{R}^2 / 2\pi \mathbf{Z}^2$, is the two dimensional torus, corresponding to periodic boundary conditions, $\Delta = \partial_{x_1}^2 + \partial_{x_2}^2$ is the Laplacian, and Δ^2 is the biharmonic operator. The model parameters γ and m are positive, β, α are coefficients of friction, and $p > 1$ is an *odd* integer. The model incorporates both external and internal linear damping terms, given by $\alpha \partial_t u(t,x)$ and $\beta \partial_t \Delta^2 u(t,x)$, respectively. System (1) describes the motion of a two dimensional beam which is subject to self

E-mail address: kosovalic@southalabama.edu.

https://doi.org/10.1016/j.jde.2018.04.022
0022-0396/© 2018 Published by Elsevier Inc.

Please cite this article in press as: N. Kosovalić, Quasi-periodic self-excited travelling waves for damped beam equations, J. Differential Equations (2018), https://doi.org/10.1016/j.jde.2018.04.022

excitation. The self excitation is reflected by the velocity dependent nonlinearity. In this paper we will show that for (γ, m) belonging to a dense subset of the positive quadrant, the combination of linear and nonlinear damping terms of opposite sign, produces a two parameter family of small amplitude quasi-periodic travelling waves. The latter are continuations of rotating wave solutions of (1), which in turn are continuations of rotating waves which arise from the linearization around zero of (1), when the friction coefficients are both zero. Hence, we obtain a codimension two bifurcation theorem, and we think of α and β as the bifurcation parameters. The choice of α and β as the bifurcation parameters is paramount. As a matter of fact, it can be seen by an energy argument that in order for (1) to have a nontrivial smooth periodic/quasi-periodic solution, either $\alpha > 0$ or $\beta > 0$. To see this let

$$E(t) = \int_{\mathbf{T}^2} \frac{1}{2}(\partial_t u)^2 + \frac{1}{2}\gamma(\Delta u)^2 + \frac{1}{2}mu^2 dxdy.$$

Then integrating by parts and exploiting periodic boundary conditions, gives

$$\begin{aligned}\frac{d}{dt}E(t) &= \int_{\mathbf{T}^2} \partial_t u[-\gamma\Delta^2 u(t,x) - \alpha\partial_t u(t,x) - \beta\partial_t\Delta^2 u(t,x) - mu + (\partial_t u)^p] + \gamma(\Delta u)(\Delta\partial_t u) \\ &\quad + mu(\partial_t u)dxdy \\ &= \int_{\mathbf{T}^2} -\alpha(\partial_t u)^2 - \beta(\partial_t u)\Delta^2\partial_t u(t,x) + (\partial_t u)^{p+1} dxdy,\end{aligned}$$

which means $E(t)$ is increasing if $\alpha, \beta \leq 0$ since p is odd and Δ^2 is positive semidefinite.

The phenomenon of self-excited vibrations is prevalent in engineering and biology. A self-excited vibration is the sustained vibration of a mechanical or biological system, arising from an external power source, which autonomously depends on the system's state. Hence, these differ from forced vibrations. In mechanical engineering self-excited vibrations occur for example in the destabilization and swaying of railway vehicles (hunting oscillations) see [8] or [12], and the shimmy of vehicle wheels, [22,23]. Since the latter are subject to damping, the positive feedback needed to sustain the oscillation in many cases arises from a time delay. In biology, self-excited vibrations are encountered in various situations ranging from neural control mechanisms, like the pupil light reflex [10,15], to hematology, [16,17]. See the survey [9] for various other examples. In contrast with forced vibrations, not much is known about self-excited vibrations for wave equations.

Although substantial work has been done on the bifurcation of periodic and quasi-periodic solutions of various types of nonlinear wave equations (including beam equations), the majority of these works focus on either on non-dissipative systems e.g. [3,5,11,2,1,24], or forced dissipative systems like in [18,19,4], and as a result do not involve any *external* bifurcation parameters in their models. On the other hand, in the works [13,14] the author and B. Pigott showed that a damped wave equation with Dirichlet boundary conditions undergoes a sequence of Hopf bifurcations when the time delay is taken as the bifurcation parameter, which seem to be the only such results for damped wave equations without forcing. In the present paper, external bifurcation parameters also play a huge role, but in contrast to these works we need two of them. Since the present system (1) is dissipative our results are more closely related to, e.g., the work by Van Gils

Please cite this article in press as: N. Kosovalić, Quasi-periodic self-excited travelling waves for damped beam equations, J. Differential Equations (2018), https://doi.org/10.1016/j.jde.2018.04.022