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J. Differential Equations 265 (2018) 1279–1323

Journal of Differential Equations

www.elsevier.com/locate/jde

L^p-estimates for the square root of elliptic systems with mixed boundary conditions

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Received 24 January 2018
Available online 16 April 2018

Abstract

This article focuses on L^p-estimates for the square root of elliptic systems of second order in divergence form on a bounded domain. We treat complex bounded measurable coefficients and allow for mixed Dirichlet/Neumann boundary conditions on domains beyond the Lipschitz class. If there is an associated bounded semigroup on L^{p_0}, then we prove that the square root extends for all $p \in (p_0, 2)$ to an isomorphism between a closed subspace of $\mathrm{W}^{1,p}$ carrying the boundary conditions and L^p. This result is sharp and extrapolates to exponents slightly above 2. As a byproduct, we obtain an optimal p-interval for the bounded H^∞-calculus on L^p. Estimates depend holomorphically on the coefficients, thereby making them applicable to questions of (non-autonomous) maximal regularity and optimal control. For completeness we also provide a short summary on the Kato square root problem in L^2 for systems with lower order terms in our setting.
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MSC: primary 35J47, 47D06, 47A60; secondary 42B20, 47B44

Keywords: Elliptic systems of second order; Mixed boundary conditions; Kato square root problem; H^∞ functional calculus; Calderón–Zygmund decomposition for Sobolev functions; Lamé system

1. Introduction and main results

Elliptic divergence form operators are amongst the most carefully studied differential operators with variable coefficients. In this paper we contribute to the functional calculus of such operators with complex, bounded and measurable coefficients, formally given by

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https://doi.org/10.1016/j.jde.2018.04.002
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$$Lu = - \sum_{i,j=1}^{d} \frac{\partial}{\partial x_i}\Big(a_{ij} \frac{\partial u}{\partial x_j}\Big) - \sum_{i=1}^{d} \frac{\partial}{\partial x_i}(a_{i0}u) + \sum_{j=1}^{d} a_{0j} \frac{\partial u}{\partial x_j} + a_{00}u, \tag{1.1}$$

on a bounded domain $\Omega \subseteq \mathbb{R}^d$, $d \geq 2$. We allow for mixed boundary conditions. Namely, u satisfies homogeneous Dirichlet boundary conditions on a closed part D of the boundary and natural boundary conditions on the complementary part $N = \partial\Omega \setminus D$. The geometric constellation can be 'rough' in that we require Lipschitz coordinate charts for $\partial\Omega$ only around the closure of N, whereas around D the domain Ω can merely be d-Ahlfors regular, and D itself has to be $(d-1)$-Ahlfors regular. These notions, henceforth called Assumptions N, Ω, and D, will be recalled in Section 2.1. We include $(m \times m)$-systems in our considerations, that is to say, u takes its values in $\mathbb{C}^m$ and each a_{ij} is valued in the space of matrices $\mathcal{L}(\mathbb{C}^m)$. As in [38], we may have different Dirichlet boundary parts for each coordinate of u. These assumptions are amongst the most general ones that allow for a proper functional analytic framework for L [6,18,38,39].

As usual, we interpret L in the weak sense via the sesquilinear form

$$a(u,v) = \int_\Omega \sum_{i,j=1}^{d} \Big(a_{ij} \frac{\partial u}{\partial x_j} \cdot \overline{\frac{\partial v}{\partial x_i}} + a_{i0}u \cdot \overline{\frac{\partial v}{\partial x_i}} + a_{0j} \frac{\partial u}{\partial x_j} \cdot \overline{v} + a_{00}u \cdot \overline{v} \Big) \, \mathrm{d}x \tag{1.2}$$

defined on $\mathcal{D}(a) = \mathbb{W}^{1,2}_D(\Omega)$, where the subscripted D is reminiscent of the boundary conditions. Ellipticity is in the sense of a Gårding inequality, turning L into a maximal accretive operator on $\mathrm{L}^2(\Omega)^m$. This way of understanding L is called 'Kato's form method'. Definitions are provided in Section 2.4 and we refer to [42,47] for the general background. Let us stress that our setup incorporates, for example, the Lamé system. We shall come back to that.

The focus in this paper lies on establishing L^p-estimates for the (unique) maximal accretive square root $L^{1/2}$ of L. More precisely, we study for which $p \in (1, \infty)$ it extends or restricts to a topological isomorphism

$$L^{1/2} : \mathbb{W}^{1,p}_D(\Omega) \xrightarrow{\cong} \mathrm{L}^p(\Omega)^m. \tag{1.3}$$

Our results are the first of this kind for 'rough' divergence form systems on domains and provide optimal ranges of exponents.

Recent years have witnessed a vast number of applications of property (1.3). It is key to the approach of Rehberg and collaborators to quasilinear parabolic equations on distribution spaces via maximal regularity techniques originating from [39] and its extensions for example to optimal control problems [17] and quasilinear stochastic evolution equations [41], as well as recent progress on maximal regularity for the non-autonomous Cauchy problem on Lebesgue spaces [34,35] and distribution spaces [23], see also [1,3,21] for the case $p = 2$. Aiming in a slightly different direction, [24] uses property (1.3) to prove Hölder continuity of solutions to quasilinear parabolic equations in rough domains.

The common idea in all of these applications is that (1.3) allows to switch between Lebesgue spaces and Sobolev spaces as well as their duals by means of an isomorphism that is build from L itself and hence commutes with the latter. This allows one to transfer knowledge between any two of these spaces. Let us give two illustrating examples. If L corresponds to an equation ($m = 1$) with real coefficients, then L has a bounded H^∞-calculus and hence maximal regularity on L^p

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