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# Existence and multiplicity of rotating periodic solutions for resonant Hamiltonian systems ☆

Guanggang Liu [a,b], Yong Li [b,c,∗], Xue Yang [b,c]

[a] School of Mathematical Sciences, Liaocheng University, Liaocheng 252000, China

[b] School of Mathematics and Statistics and Center for Mathematics and Interdisciplinary Sciences, Northeast Normal University, Changchun 130024, China

[c] College of Mathematics, Jilin University, Changchun 130012, China

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**Abstract**

In the present paper, we consider a class of resonant Hamiltonian systems $x' = JH_x(t,x)$ in $\mathbb{R}^{2N}$. We will use saddle point reduction, Morse theory combining the technique of penalized functionals to obtain the existence of nontrivial rotating periodic solutions, i.e., $x(t+T) = Qx(t)$ for any $t \in \mathbb{R}$ with $T > 0$ and $Q$ an symplectic orthogonal matrix. In the case: $Q^k \neq I_{2N}$ for any positive integer $k$, such a rotating periodic solution is just a quasi-periodic solution. Moreover, if $H$ is even in $x$, we will give the multiplicity of nontrivial rotating periodic solutions by using two abstract critical theorems and previous techniques.


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*Keywords:* Asymptotically linear Hamiltonian systems; Rotating periodic solutions; Morse theory

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∗ Corresponding author at: School of Mathematics and Statistics and Center for Mathematics and Interdisciplinary Sciences, Northeast Normal University, Changchun, 130024, China.

*E-mail addresses:* lgg112@163.com (G. Liu), liyongmath@163.com (Y. Li), yangxuemath@163.com (X. Yang).

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## 1. Introduction

In this paper we consider the following Hamiltonian system

$$x' = JH_x(t,x), \tag{1.1}$$

where $x \in \mathbb{R}^{2N}$, $J = \begin{pmatrix} 0 & -I_N \\ I_N & 0 \end{pmatrix}$, $I_N$ is the $N$-dimensional identity matrix, $H(t,x) \in C^2(\mathbb{R} \times \mathbb{R}^{2N}, \mathbb{R})$ with $H(t+T,x) = H(t, Q^{-1}x)$ for some $2N \times 2N$ orthogonal symplectic matrix $Q$. We will show that (1.1) has solutions of the form $x(t+T) = Qx(t), \forall t \in \mathbb{R}$. This type of solutions are called the rotating periodic solutions of (1.1). When $Q = I_{2N}$, $I_{2N}$ denotes the identity matrix in $\mathbb{R}^{2N}$, this type of rotating periodic solutions are familiar periodic solutions. When $Q^k = I_{2N}$ for some $k \in \mathbb{Z}^+$ with $k \geq 2$, this type of rotating periodic solutions are subharmonic solutions. When $Q^k \neq I_{2N}$ for any $k \in \mathbb{Z}^+$ with $k \geq 1$, this type of rotating periodic solutions are quasi-periodic solutions.

Since the study by Poincaré on three body problems, the periodic solutions of Hamiltonian systems have been extensively studied. In the last three decades, the variational methods have been widely used in the existence and multiplicity of periodic solutions for Hamiltonian systems under various suitable solvability conditions (see [2,6,12,13,18,21,23,24,26,28,29,32–34,37,38,40] and references therein). Recently, the stability and existence of rotating periodic solutions have attracted much of people's interest and attention. In [15,16], using the Maslov index theory, Hu et al. built up some important stability criteria for rotating periodic solutions of Hamiltonian systems. In [9,10], Chang and Li studied the existence of rotating periodic solutions for the second order dynamical systems by using the coincidence degree theory. In [22], by using Morse theory and the technique of penalized functional, we obtained the existence of nontrivial rotating periodic solutions for a class of asymptotically linear second order Hamiltonian systems with resonance at infinity.

We will consider asymptotically linear Hamiltonian systems in this paper. If the asymptotical operator of the Hamiltonian system at infinity is degenerate, we call this system is resonant at infinity. When the system is resonant at infinity, one main difficulty arises from the lack of compactness. To overcome this difficulty, some additional technical conditions such as Landesmann–Lazer condition, the strong resonant conditions are usually imposed on the nonlinearity, see [4,8,18,19,39] and references therein. These technical assumptions ensure that the functional satisfies the (PS) condition in global or apart from some exceptional levels. In [6], Benci and Fortunato considered a new resonant condition for second order Hamiltonian systems. They consider the following system

$$-x'' = A(t)x + G_x(x,t), \tag{1.2}$$

where $A(t)$ is a symmetric $T$ periodic $n \times n$ matrix, and

$$G(x,t) \quad \text{and} \quad G_x(x,t) \quad \text{are bounded,} \quad \lim_{|x|\to\infty} |G_{xx}(x,t)| = 0. \tag{1.3}$$

The resonant conditions (1.3) imply that the (PS) condition may fail at any level, to overcome this difficulty, they proposed the technique of penalized functionals and obtained the existence of nontrivial periodic solution by combining Morse theory. Later, the similar resonant conditions

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