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J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Homogenization of the compressible Navier–Stokes equations in domains with very tiny holes

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Received 7 April 2017; revised 13 March 2018

Abstract

We consider the homogenization problem of the compressible Navier–Stokes equations in a bounded three dimensional domain perforated with very tiny holes. As the number of holes increases to infinity, we show that, if the size of the holes is small enough, the homogenized equations are the same as the compressible Navier–Stokes equations in the homogeneous domain—domain without holes. This coincides with the previous studies for the Stokes equations and the stationary Navier–Stokes equations. It is the first result of this kind in the instationary barotropic compressible setting. The main technical novelty is the study of the Bogovskiĭ operator in non-Lipschitz domains.

Keywords: Homogenization; Navier–Stokes equations; Perforated domains; Bogovskiĭ operator

1. Introduction

In practice, there comes up the study of fluid flows in domains distributed with a large number of *holes* that represent solid obstacles. It is suitable, for instance, to model polluted underground water or oil development. The fluid flows passes between the small obstacles or in the holes in between. Such domains are usually called perforated domains and a typical example is the so-called porous media. The perforation parameters, which are mainly the size of holes and the

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https://doi.org/10.1016/j.jde.2018.04.007
0022-0396/

mutual distance of the holes in the perforated domain under consideration, play a determinant role in these problems. We refer to [22] for a number of real world applications.

Homogenization problems in fluid mechanics represent the study of the asymptotic behavior of fluid flows in perforated domains as the number of holes (obstacles) goes to infinity and the size of holes (obstacles) goes to zero simultaneously. The mathematical concern is the asymptotic behavior of the solutions to equations describing fluid flows with respect to perforation parameters. With an increasing number of holes within the domain of the fluid, the fluid flow approaches an effective state governed by certain *homogenized* equations in the full domain.

With different physical backgrounds, the mathematical equations governing the fluid flows are different. There are typically Stokes equations, Navier–Stokes equations or Euler equations. There also include equations describing non-Newtonian fluid flows, such as p-Stokes equations, Oldroyd-B models, and many others. Accordingly, the mathematical study of homogenization problems in fluid mechanics includes the homogenization for various fluid models. We refer to [24,2,3] for Stokes equations, [20] for incompressible Navier–Stokes equations, [19] for compressible Navier–Stokes equations and [13] for the complete compressible Navier–Stokes–Fourier equations. We also refer to the book [15] for other models, such as two phase models and non-Newtonian fluid models.

In this paper, we study the homogenization problem for the compressible Navier–Stokes equations. We consider a bounded three dimensional domain perforated with tiny holes, where the diameters of the holes are taken to be of size $O(\varepsilon^\alpha)$ with $\alpha \geq 1$, and the minimal mutual distances between the holes are of size $O(\varepsilon)$. In this work we will provide a size parameter $\alpha_0 \geq 1$ to be specified later on in Theorem 1.6, such that for $\alpha \geq \alpha_0$ a homogenization sequence of solutions converges to the solution of the compressible Navier–Stokes system in the non-perforated domain. This means that if the size of the holes is small enough, then in the homogenization limit they shall not be seen anymore. This is a known phenomenon: analogous results for stationary incompressible or compressible fluids have been shown and will be discussed below in more detail. However, we would like to emphasize that up to our knowledge, this result is the first one in this direction for unsteady compressible fluids.

Let us introduce the setting in more detail. Let $\Omega \subset \mathbb{R}^3$ be a C^2 bounded domain and $\{T_{\varepsilon,k}\}_{k\in K_\varepsilon} \subset \Omega$ be a family of closed sets (named *holes* or *solid obstacles*) satisfying

$$T_{\varepsilon,k} = x_{\varepsilon,k} + \varepsilon^\alpha T_k \subset B(x_{\varepsilon,k}, \delta_0\varepsilon^\alpha) \subset B(x_{\varepsilon,k}, \delta_1\varepsilon^\alpha) \subset B(x_{\varepsilon,k}, \delta_2\varepsilon) \subset B(x_{\varepsilon,k}, \delta_3\varepsilon) \subset \Omega, \quad (1.1)$$

where for each k, $T_k \subset \mathbb{R}^3$ is a simply connected bounded domain of class C^2, where the C^2 constant is assumed to be uniformly bounded in k. With $B(x,r)$ we denote the open ball centered at x with radius r in $\mathbb{R}^3$. Here we assume that $\delta_0,\ \delta_1,\ \delta_2,\ \delta_3$ are positive fixed constants independent of ε, such that $\delta_0 < \delta_1$ and $\delta_2 < \delta_3$. Moreover, we suppose that the balls (control volumes) $\{B(x_{\varepsilon,k}, \delta_3\varepsilon)\}_{k\in K_\varepsilon}$ are pairwise disjoint. The corresponding ε-dependent perforated domain is then defined as

$$\Omega_\varepsilon := \Omega \setminus \bigcup_{k\in K_\varepsilon} T_{\varepsilon,k}. \quad (1.2)$$

By the distribution of holes assumed above, the number of holes in Ω_ε satisfy

$$|K_\varepsilon| \leq C\frac{|\Omega|}{\varepsilon^3}, \quad \text{for some } C \text{ independent of } \varepsilon. \quad (1.3)$$

Please cite this article in press as: Y. Lu, S. Schwarzacher, Homogenization of the compressible Navier–Stokes equations in domains with very tiny holes, J. Differential Equations (2018), https://doi.org/10.1016/j.jde.2018.04.007