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Ergodic attractors and almost-everywhere asymptotics of scalar semilinear parabolic differential equations

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Abstract

We consider dynamics of scalar semilinear parabolic equations on bounded intervals with periodic boundary conditions, and on the entire real line, with a general nonlinearity $g(t, x, u, u_x)$ either not depending on t, or periodic in t. While the topological and geometric structure of their attractors has been investigated in depth, we focus here on ergodic-theoretical properties. The main result is that the union of supports of all the invariant measures projects one-to-one to $\mathbb{R}^2$. We rely on a novel application of the zero-number techniques with respect to evolution of measures on the phase space, and on properties of the flux of zeroes, and the dissipation of zeroes. As an example of an application, we prove uniqueness of an invariant measure for a large family of considered equations which conserve a certain quantity ("mass"), thus generalizing the results by Sinai for the scalar viscous Burgers equation.

MSC: primary 35K15, 37L40; secondary 35B40, 35B41, 37L30

Keywords: Scalar semilinear parabolic equations; Burgers equation; Reaction–diffusion equation; Attractor; Invariant measure; Zero number

Contents

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Part 1. Preliminaries

1. Introduction

We consider the following equation:

$$u_t = u_{xx} + g(t, x, u, u_x), \tag{1.1}$$

where g satisfies the usual conditions guaranteeing local existence of solutions, given as (A1-3) below. In particular, we assume that g is periodic in t, x, and consider solutions on the entire real line, without the assumption of decay to 0 at infinity (the *extended, time-periodic case*). For the sake of completeness, we also cover less general cases of g not depending on t (the *autonomous case*), and for $x \in \mathbb{S}^1$ (the *bounded case*). A more precise setting is given below. For brevity, we frequently denote the considered cases with letters E or B (for extended vs. bounded), and P or A (time-periodic vs. autonomous).

We first briefly recall results on geometric and topological dynamics of (1.1). The asymptotics of (1.1) on the bounded domain with separated boundary conditions has been studied in detail (see [21,27] and references therein) and is reasonably well-understood. In particular, under assumptions (A1-3), for any global, uniformly bounded orbit, the ω-limit set contains a single orbit (equilibria in the autonomous or a periodic orbit in the periodic case) ([27], Theorem 4.2 and references therein). With periodic boundary conditions, i.e. in our setting in the B/A case and assuming (A1-3), Fiedler and Mallet-Paret [12] have shown that the ω-limit set of any global, bounded solution projects to a plane, and then has the structure in accordance to the Poincaré–Bendixson theorem. That means that it consists of a single periodic orbit, or of equilibria and connecting (homoclinic and heteroclinic) orbits. Tereščák [39] has shown that in the B/P case,

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