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On a parabolic–elliptic system with gradient dependent chemotactic coefficient ☆

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Abstract

We consider a second order PDEs system of Parabolic–Elliptic type with chemotactic terms. The system describes the evolution of a biological species "u" moving towards a higher concentration of a chemical stimuli "v" in a bounded and open domain of $\mathbb{R}^N$. In the system considered, the chemotaxis sensitivity depends on the gradient of v, i.e., the chemotaxis term has the following expression

$$-div\left(\chi u|\nabla v|^{p-2}\nabla v\right),$$

where χ is a positive constant and p satisfies

$$p \in (1, \infty), \quad \text{if } N = 1 \quad \text{and} \quad p \in \left(1, \frac{N}{N-1}\right), \quad \text{if } N \geq 2.$$

We obtain uniform bounds in $L^\infty(\Omega)$ and the existence of global in time solutions. For the one-dimensional case we prove the existence of infinitely many non-constant steady-states for $p \in (1, 2)$ for any χ positive and a given positive mass.

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1. Introduction

Chemotaxis is the ability of some living organisms to orient their movement along a chemical concentration gradient. The process has been extensively studied from a biological point of view after the development of the microscope during the XIX century. In the last decades, several mathematical models have been presented to describe the phenomenon, after the pioneering works of Patlak [28] and Keller and Segel [22], [23] (see also the review articles Horstmann [19], [20] and Bellomo et al. [3] and references therein for more extensive literature in the subject). The original model in [22] describes the evolution of a biological species, denoted by "u" in terms of a parabolic equation, with linear diffusion and a second order nonlinear term in the form

$$-div(\chi u \nabla v),$$

where v denotes the concentration of the chemical stimuli.

In the last years, linear diffusion of the biological species "u" has been replaced in different ways:

- by nonlinear diffusion at the form $-div(\phi(u)\nabla u)$, see for instance Wrzosek [36], Cieslak and Morales-Rodrigo [16], Cieslak and Winkler [17], Winkler [35].
- by fractional diffusion, see J. Burczak, R. Granero-Belinchón [13] and [14] among others.
- by nonlinear diffusion depending on $|\nabla u|^p$, (p-laplacian), see Bendahmane [6].

The system has been also studied for several biological species, see for instance Tang and Tao [31], Tello and Winkler [33], Stinner, Tello and Winkler [30], Negreanu and Tello [26] and [27], Wang and Wu [34] among others. In the last years, several mathematical models have considered the chemotactic sensitivity coefficient "χ" dependent on ∇v instead of constant. For instance, in Bellomo and Winkler [4] and [5] (see also Bellomo et al. [2]), a chemotaxis system is analyzed for a chemotactic term of the form

$$-div\left(\frac{\chi u}{[1+|\nabla v|^2]^{\frac{1}{2}}}\nabla v\right).$$

In Bianchi, Painter and Sherratt [7] the authors consider the term

$$-div\left(\frac{\chi u}{(1+\omega u)}\frac{\nabla v}{(1+\eta|\nabla v|)}\right),$$

for some positive constants χ, ω and η. In [7], the authors study a system of four PDEs of parabolic type in an one-dimensional spatial domain coupled with an ODE modeling Lymphangiogenesis in wound healing.

A general chemotactic term is presented as

$$-div\left[u\tilde{\chi}(u,v,|\nabla v|)\nabla v\right],$$

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