ARTICLE IN PRESS YJDEQ:9251

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J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

A heteroclinic orbit connecting traveling waves pertaining to different nonlinearities

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Received 13 December 2017

Abstract

In this paper we consider a semilinear parabolic equation in an infinite cylinder. The spatially varying nonlinearity is such that it connects two (spatially independent) bistable nonlinearities in a compact set in space. We prove that, given such a setting, a traveling wave obeying the equation with the one bistable nonlinearity and starting at the respective side of the cylinder, will converge to a traveling wave solution prescribed by the nonlinearity on the other side.

1. Introduction

Since the pioneering works of Kolmogorov et al. [8] the study of traveling wave solutions for semilinear parabolic equations of several types (the most prominent are bistable, ignition and KPP-type nonlinearities) has been an active field. In the case of bistable nonlinearities, that we will concern ourselves with in this article, we want to refer to the celebrated paper by Fife and McLeod [6] for existence and uniqueness as well as stability in one spatial dimension. We also want to mention the detailed study of traveling waves in cylinders by Berestycki and Nirenberg [4]. Later Berestycki, Hamel et al. have broadened the field by studying generalizations of traveling waves in domains or with coefficients / nonlinearities that do not allow for traveling wave solutions. In the case of periodic media, this has led to the notion of pulsating fronts [1] and recently they have generalized it to the notion of transition fronts that do not require any

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https://doi.org/10.1016/j.jde.2018.03.007

Please cite this article in press as: S. Eberle, A heteroclinic orbit connecting traveling waves pertaining to different nonlinearities, J. Differential Equations (2018), https://doi.org/10.1016/j.jde.2018.03.007

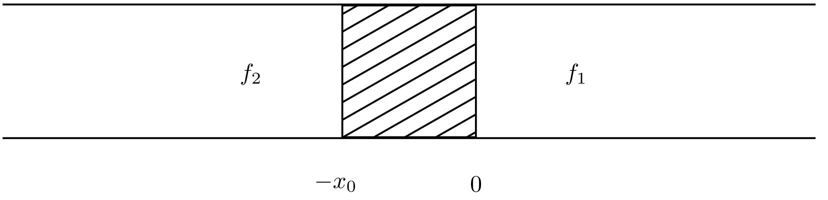

Fig. 1. Infinite cylinder with transition zone.

special properties of the domain (apart from sufficiently smooth boundary and infinite geodesic diameter) or of the coefficients [2]. In this respect we found [3] very inspiring where Matano, Berestycki and Hamel use super- and subsolutions as in [9] to construct an entire solution that starts as a traveling wave solution for $t \to -\infty$, passes the obstacle and converges – given the compact obstacle is sufficiently regular – to the same traveling wave solution as $t \to +\infty$. Another recent and very interesting work on the construction of generalized transition fronts is [10] where the author studies generalized transition fronts in cylinders subject to a space dependent nonlinearity that is bounded form above and below by spatially independent ignition-type nonlinearities. We are trying to investigate a similar problem as is investigated in [10], but in our case the nonlinearities are of bistable type and do only vary in a compact transition zone. In contrast to [10] we are interested in the existence of a heteroclinic connection between two traveling fronts, which is stronger than the very relaxed notion of a transition front (as it is given in [2]) and we apply different methods.

In this paper we will occupy ourselves with the construction of a transition front in a cylinder $D = \mathbb{R} \times \Omega$. But thanks to the compactness of the transition region, we can construct a heteroclinic orbit between two traveling wave solutions. To be more precise the nonlinearity $f(x, u)$ shall be such that

$$\begin{cases} f_2(u) \leq f(x,u) \leq f_1(u) \text{ for } x \in D, u \in [0,1], \\ f(x,u) = f_1(u) \text{ for } x_1 \geq 0, u \in [0,1] \text{ and} \\ f(x,u) = f_2(u) \text{ for } x_1 \leq -x_0, u \in [0,1], \end{cases} \tag{1.1}$$

where f_1, f_2 are two a-priori given nonlinearities of bistable type and $x_0 > 0$ is a parameter of the transition region (see Fig. 1).

The main result of this article shall be

Theorem 1.1. *Let f satisfy (1.1) (and (2.1)–(2.5)) then there is a unique entire solution $u(t, x)$ of*

$$\begin{cases} \partial_t u - \Delta u = f(x,u) & \text{in } D, \\ \frac{\partial u}{\partial \nu} = 0 & \text{on } \partial D, \end{cases} \tag{1.2}$$

such that $0 < u(t,x) < 1$ and $\partial_t u(t,x) > 0$ for all $(t,x) \in \mathbb{R} \times \bar{D}$ and

$$u(t,x) - \phi_1(x_1 + c_1 t) \to 0 \text{ as } t \to -\infty \text{ uniformly in } x \in \bar{D},$$

then

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