ARTICLE IN PRESS YJDEQ:9258

Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Initial-boundary value problem of a parabolic–hyperbolic system arising from tumor angiogenesis

Shijin Deng

School of Mathematical Sciences, Shanghai Jiao Tong University, China

Received 4 January 2018; revised 2 March 2018

Abstract

In this paper, we consider the half-space problem for a parabolic–hyperbolic system arising from tumor angiogenesis. Under a mixed type boundary condition, we construct the Green's function for half-space problem by relating it with fundamental solution for an initial value problem. After differential equation method for the boundary operator and proper estimates for additional exponential factors generated by the asymmetry of characteristics, one reduces the Green's function (for initial-boundary value problem) into fundamental solution (for Cauchy problem) which could be estimated by singularity removal, long wave-short wave decomposition and weighted energy method outside cone. We finally obtain the pointwise estimate for Green's function which results in the pointwise convergence rate for nonlinear problem. The Green's function constructed is precise enough to avoid a priori estimate for derivatives by energy method. The half space problem studied here is a main ingredient for future study of shock profile with presence of boundary.

Keywords: Parabolic–hyperbolic system; Initial-boundary value problem; Green's function method; Recombination; Boundary operator; Pointwise structure

E-mail address: matdengs@sjtu.edu.cn.

https://doi.org/10.1016/j.jde.2018.03.014
0022-0396/© 2018 Published by Elsevier Inc.

1. Introduction

The following parabolic–hyperbolic system:

$$\begin{cases} u_t - \chi(uv)_x = Du_{xx}, \\ v_t - u_x = 0, \end{cases} \tag{1.1}$$

is obtained from the PDE–ODE hybrid model

$$\begin{cases} u_t = (Du_x - \xi u(\ln c)_x)_x, \\ c_t = -\mu uc \end{cases} \tag{1.2}$$

after a Cole–Hopf type transformation

$$v = -\frac{1}{\mu}(\ln c)_x, \tag{1.3}$$

with $\chi \equiv \xi\mu$. In the biological model (1.2), the unknown functions $u(x,t)$ and $c(x,t)$ denote the density of vascular endothelial cells and concentration of vascular endothelial growth factor respectively. In (1.2), the parameter $D > 0$ is the diffusivity of endothelial cells, $\xi > 0$ is the chemotactic coefficient measuring the intensity of chemotaxis and μ denotes the degradation rate of the chemical c. More backgrounds and the treatment of the singular logarithmic sensitivity in (1.2) by the Cole–Hopf transformation (1.3) could be found in [1,11–13,15,16,30,32].

In this paper, we consider the system (1.1) in half space $x > 0$. We are interested in the boundary effect when the initial functions are small perturbations of the constant state (u_+, v_+), which satisfies

$$u_+ > 0, \quad v_+ \le 0 \tag{1.4}$$

for the biological meaning coming from the original system (1.2). We denote the initial functions as follows:

$$(u, v)(x, 0) \equiv (u_0, v_0)(x), \tag{1.5}$$

and a mixed type boundary condition is posed:

$$a_1 u(0,t) + a_2 u_x(0,t) = a_1 u_+. \tag{1.6}$$

Most previous works for this chemotaxis model are considered in whole space or bounded domain. For the bounded domain, attractive and repulsive chemotaxis are considered in [8,18,31,34]. In [34], global existence of the small perturbation for (1.1) on the bounded interval (0, 1) under Dirichlet boundary condition is obtained. In [18,31], the viscous system for repulsive chemotaxis (adding viscous term in the second equation of (1.2) and with $\xi < 0$) is considered on (0, 1) under Dirichlet boundary condition and Neumann boundary condition and both existence of global solutions and exponentially decaying rates of the solutions are obtained for large perturbation problem. In [8], (1.2) with $\xi < 0$ and its viscous system are considered on (0, 1) under

Please cite this article in press as: S. Deng, Initial-boundary value problem of a parabolic–hyperbolic system arising from tumor angiogenesis, J. Differential Equations (2018), https://doi.org/10.1016/j.jde.2018.03.014