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Measure topologically stable flows

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Abstract

Topological stability is a kind of stability for given dynamical systems in which continuous perturbations are allowed. Very recently, the concept of topological stability for Borel measures with respect to a given homeomorphism was introduced by Lee and Morales in [7]. In this paper, we introduce a notion of measure topological stability for a continuous flow, and prove that if every measure expansive flow has measure shadowing property then it is measure topologically stable.

MSC: 37C75; 54H20; 49J53

Keywords: Measure shadowing; Measure topological stability; Stability theory

1. Introduction

The concept of topological stability for a homeomorphism on a compact metric space introduced by Walters [12] is important in the qualitative study of dynamical systems. Walters proved that Anosov diffeomorphisms on closed manifolds are not only structural but also topologically stable. And he proved that the expansiveness and the shadowing property play an important role in the study of topological stability of maps on a compact metric space in [13]. Several interesting properties of topological stability have been obtained elsewhere [5,6,9,10,14]. In light of the rich consequence of topological stability in the dynamics of a system, it is natural to consider

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another notion of topological stability. Very recently, Lee and Morales [7] introduced the notion of *measure topological stability*, generalizing the usual concept of topological stability and the new result; every expansive measure with the shadowing property is topologically stable with respect to a homeomorphism.

On the other hand, Thomas [11] proposed an extension of topological stability to flow and proved that every continuous expansive flow without fixed points which has the shadowing property is topologically stable. This is the extended result for continuous flows of Walters' one [13]. And Carrasco-Olivera and Morales [4] proposed an extension of measure expansiveness using Borel measures on a compact metric space to continuous flows and proved that there were no measure expansive flows on closed surfaces.

In this paper we introduce concepts of topological stability and shadowing property for Borel measures with respect to a continuous flow. And we prove that every measure expansive flow with the measure shadowing property on a compact metric space is measure topologically stable. This extends the results of Lee et al. [7] which were obtained for the case of homeomorphisms to the case of continuous flows. Moreover we investigate the relationship between topologically stable measures for homeomorphisms and their corresponding suspensions.

2. New definitions

Let (X, d) be a compact metric space. A *flow* on X is a continuous map $\phi : X \times \mathbb{R} \to X$ satisfying $\phi(x, 0) = x$ and $\phi(\phi(x, s), t) = \phi(x, s + t)$ for $x \in X$ and $s, t \in \mathbb{R}$. For convenience, we denote

$$\phi(x, s) = \phi_s(x) \quad \text{and} \quad \phi_{(a,b)}(x) = \{\phi_t(x) : t \in (a, b)\}.$$

The set $\mathcal{O}_\phi(x) = \phi_{\mathbb{R}}(x)$ is called the *orbit of ϕ through $x \in X$*.

In this section, we introduce new definitions for continuous flows on a compact metric space from a measure theoretic viewpoint.

2.1. Measure shadowable flows

The notion of shadowing property plays an important role in the study of topological properties in dynamical systems. It is well known that a topologically stable homeomorphism has the shadowing property.

Let us recall the definition of shadowing property for continuous flows in [8]. We say that a sequence $\{(x_i, t_i) : x_i \in X, t_i \geq 1, -\infty \leq a < i < b \leq \infty\}$ is *a $(\delta, 1)$-pseudo orbit* of ϕ if for any $a < i < b - 1$,

$$d(\phi_{t_i}(x_i), x_{i+1}) < \delta.$$

We say that a flow ϕ is *shadowable* if for any $\varepsilon > 0$, there is $\delta > 0$ satisfying the following property: given any $(\delta, 1)$-pseudo orbit, there exist a point $y \in X$ and a continuous map $\tau : X \times \mathbb{R} \to \mathbb{R}$ such that $\tau(x, \cdot) : \mathbb{R} \to \mathbb{R}$ is strictly increasing homeomorphism with $\tau(x, 0) = 0$ and

$$d(\phi_{\tau(y,t)}(y), \phi_{t-T_i}(x_i)) < \varepsilon, \quad T_i \leq t < T_{i+1},$$

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