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Traveling wave solutions of a time-periodic competitive system with a free boundary ☆

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Abstract

We consider a time-periodic competition system with a free boundary, which can be used to describe two species that are struggling on the free boundary to obtain their own habitats and the final competition result is the regional partition of two species. In [15], we have studied the (classical) traveling wave solutions of the temporal homogeneous problems. In this paper, we consider the time-periodic case and study periodic traveling wave solutions of the problem. More precisely, we will prove the existence and uniqueness of the periodic traveling wave solution, and show that the speed of the periodic traveling wave is uniquely determined by and monotonically dependent on the time-periodic terms in the system. Moreover, we will present some sufficient conditions to guarantee that the wave travels with a positive speed.

MSC: 35C07; 35K57; 35R35

Keywords: Traveling wave solution; Time-periodic competitive system; Free boundary problem

1. Introduction

In population ecology, the appearance of regional partition of multi-species through strong competition is one interesting phenomenon. Some authors used the following reaction–diffusion equation system

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$$\begin{cases} u_t = d_1 u_{xx} + f(u), & x < S(t),\ t > 0, \\ v_t = d_2 v_{xx} + g(v), & x > S(t),\ t > 0, \\ S'(t) = -\alpha u_x(t,x) - \beta v_x(t,x), & x = S(t),\ t > 0, \\ u(t,x) = v(t,x) = 0, & x = S(t),\ t > 0, \\ S(0) = 0,\ u(0,x) = u_0(x), & x \le 0, \\ v(0,x) = v_0(x), & x \ge 0, \end{cases} \tag{1.1}$$

to describe regional partition of two species, where $x = S(t)$ is the free boundary to be determined together with u and v; $d_1,\ d_2,\ \alpha$ and β are given positive constants. $f,\ g : [0,\infty) \to \mathbb{R}$ are C^1 functions satisfying $f(0) = g(0) = 0$. Actually, the two-phase Stefan condition $(1.1)_3$ (hereafter, we use $(1.1)_3$ to denote the third equation in (1.1)) can be derived as a spatial segregation limit of some strong competition systems. Namely, when we take singular limit as the interspecific competition rate goes to infinity in certain competition systems, the two-phase Stefan free boundary which separates one competitor from another will appear (cf. [2,3,7,11,14] etc.).

In 1980s', Mimura, Yamada and Yotsutani [8–10] considered the problem (1.1) in the bounded domain [0, 1], with Dirichlet boundary conditions and $S(t) \in [0,1]$. They studied the global existence, uniqueness, regularity and the asymptotic behavior of the solutions. Their argument is based on the notion of ω-limit set and the comparison principle. In particular, when the homogenous Dirichlet boundary conditions are imposed, in [10], they found an interesting phenomenon, that is, the free boundary $S(t)$ may hit one of the ends $x = 0, 1$ in a finite time, and then it will stay there for ever. As a result, one phase in $(1.1)_3$ will disappear.

When the problem (1.1) is considered in $(-\infty,\infty)$, traveling wave solutions become a kind of interesting solutions. Let (ϕ,ψ,c) be a solution of the following problem

$$\begin{cases} d_1\phi'' + c\phi' + f(\phi) = 0, & x \in (-\infty, 0], \\ d_2\psi'' + c\psi' + g(\psi) = 0, & x \in [0,\infty), \\ \phi(0) = \psi(0) = 0, \\ \phi(-\infty) = \psi(+\infty) = 1, \\ c = -\alpha\phi'(0) - \beta\psi'(0), \end{cases} \tag{1.2}$$

then $(u,v,S) = (\phi(x-ct), \psi(x-ct), ct)$ is a traveling wave solution of the problem (1.1). Recently, in [15], the authors investigated the existence and uniqueness of the solution of (1.2) for general nonlinearities. They proved that for any given $d_1,\ d_2,\ \alpha > 0$ and for any c in a certain interval depending on $f,\ g$ and α, there exists a unique $\beta(c) > 0$ such that (1.2) has a unique solution (ϕ,ψ,c). Moreover, $\beta(c)$ is continuous and strictly decreasing in c.

The above mentioned studies are all considered in homogenous environment. However, in the field of ecology, the species generally live in heterogenous environment, which changes with time or season. The simplest example of such environments is time-periodic ones. Recently, some authors studied the time-periodic reaction–diffusion equations with free boundaries. For example, in [4,12,13], the authors considered a (single) diffusive logistic equation in time-periodic environment with a one-phase Stefan condition. In the present paper, we intend to study the system (1.1) with two-phase Stefan condition in time-periodic environment:

$$\begin{cases} u_t = d_1 u_{xx} + f(t,u), & x < S(t),\ t > 0, \\ v_t = d_2 v_{xx} + g(t,v), & x > S(t),\ t > 0, \\ S'(t) = -\alpha(t) u_x(t,x) - \beta(t) v_x(t,x), & x = S(t),\ t > 0, \\ u(t,x) = v(t,x) = 0, & x = S(t),\ t > 0, \end{cases} \tag{1.3}$$

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