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# Normalized solutions and asymptotical behavior of minimizer for the coupled Hartree equations

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**Abstract**

We prove the existence and nonexistence of $L^2(\mathbb{R}^N)$-normalized solutions of coupled Hartree equations, which arisen from the studies of the nonlinear optics and multi-component Bose–Einstein condensates. Under certain type trapping potentials, by proving some delicate energy estimates, we give a precise description on the concentration behavior of minimizer solutions of the system. Furthermore, an optimal blowing up rate for the minimizer solutions of the system is also proved.


*MSC:* 35J61; 35J20; 35Q55; 49J40

*Keywords:* Coupled Hartree equations; Nonlocal interaction; Variational methods

## 1. Introduction and main results

In the present paper we study the coupled nonlinear Hartree equations with nonlocal interaction in the following form:

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$$\begin{cases} -\Delta u + V_1(x)u = \lambda_1 u + \mu_1 \left( \int\limits_{\mathbb{R}^N} \frac{u^2(y)}{|x-y|^2} dy \right) u + \beta \left( \int\limits_{\mathbb{R}^N} \frac{v^2(y)}{|x-y|^2} dy \right) u, & x \in \mathbb{R}^N, \\ -\Delta v + V_2(x)v = \lambda_2 u + \mu_2 \left( \int\limits_{\mathbb{R}^N} \frac{v^2(y)}{|x-y|^2} dy \right) v, +\beta \left( \int\limits_{\mathbb{R}^N} \frac{u^2(y)}{|x-y|^2} dy \right) v, & x \in \mathbb{R}^N, \end{cases} \tag{1.1}$$

where $u, v \in H^1(\mathbb{R}^N)$, $\mu_1, \mu_2 > 0$, $\lambda_1, \lambda_2 \in \mathbb{R}$ are the Lagrange multipliers, $V_1(x)$ and $V_2(x)$ are trapping potentials, and $\beta \in \mathbb{R}$ is a coupling constant describing attractive or repulsive interactions.

The consideration of (1.1) is mainly motivated by recent studies on the nonlinear Hartree equation (NHE)

$$i\frac{\partial \psi}{\partial t} = -\frac{1}{2}\Delta\psi + V\psi - \chi\left(C(x) * |\psi|^2\right)\psi, \;\; x \in \mathbb{R}^3. \tag{1.2}$$

For identical and nonrelativistic basic particles (such as bosons or electrons) under the influence of an external potential and also two-body attractive interaction between two particles, the condensate in the mean field regime is governed by the NHE (see [9,10,12,16]). In (1.2), $\psi$ is a radially symmetric two-body potential function defined in $\mathbb{R}^3$ and $*$ denotes the convolution in $\mathbb{R}^3$. The most typical external potential is the Coulomb function $C(x) = |x|^{-1}$. The equation (1.2) is also used in the description of the Bose–Einstein condensates, in which $V$ is the trapping potential and the nonlocal interaction also describes the interaction between the bosons in the condensate [7,33,34]. When $V = 1$, the equation (1.2) is also known as the nonlinear Choquard equation (see [19,22,25]), and the equation (1.2) also arises from the model of wave propagation in a media with a large response length [1,17]. Recently, the papers [11,25,29,31] considered the stationary solution of (1.2) with the following generalized Choquard equation

$$-\Delta u + u = (J_\alpha * |u|^p)|u|^{p-2}u, \;\; u \in H^1(\mathbb{R}^N), \tag{1.3}$$

where $J_\alpha : \mathbb{R}^N \to \mathbb{R}$ is the Riesz potential defined at each point $x \in \mathbb{R}^N \setminus \{0\}$ by

$$J_\alpha(x) = \frac{A_\alpha}{|x|^{N-\alpha}} \quad \text{and} \quad A_\alpha = \frac{\Gamma\left(\frac{N-\alpha}{2}\right)}{\Gamma\left(\frac{\alpha}{2}\right)\pi^{N/2}2^\alpha}. \tag{1.4}$$

By using the moving plane method for the nonlocal problem (see [5,6]), Ma and Zhao [25] considered the existence and the uniqueness of the positive solutions of (1.3). By using the variational methods, Moroz and Van Schaftingen [29] proved the existence of the ground state solution of (1.3). Later on, Moroz and Van Schaftingen studied the existence of nontrivial solution of (1.3) with the lower critical case $p = \frac{\alpha}{N} + 1$ in [31], subsequently Ghimenti and Van Schaftingen proved the existence of nodal and sign-changing solutions of (1.3) in [11]. One can see [30] for general nonlinearity case.

Systems of coupled nonlinear Schrödinger equations or Hartree equations have been the focus of many recent theoretical studies. The two-component nonlinear Schrödinger equations system with nonlocal Hartree type interaction can be written in the following form:

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