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Convergence of the solution of the stochastic 3D globally modified Cahn–Hilliard–Navier–Stokes equations

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Abstract

We study in this article the stochastic 3D globally modified Cahn–Hilliard–Navier–Stokes model in a 3D dimensional bounded domain. We prove the existence and uniqueness of strong solutions. Furthermore, we discuss the relation of the stochastic 3D globally modified Cahn–Hilliard–Navier–Stokes equations to the stochastic 3D Cahn–Hilliard–Navier–Stokes equations by proving a convergence theorem, that as the parameter N tends to infinity, a subsequence of solutions of the stochastic 3D globally modified Cahn–Hilliard–Navier–Stokes equations converges to a weak martingale solution of the stochastic 3D Cahn–Hilliard–Navier–Stokes equations.

MSC: 35R60; 35Q35; 60H15; 76M35; 86A05

Keywords: Cahn–Hilliard–Navier–Stokes; Globally modified; Stochastic; Galerkin scheme

1. Introduction

It is well accepted that the incompressible Navier–Stokes (NS) equation governs the motions of single-phase fluids such as air or water. On the other hand, we are faced with the difficult

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problem of understanding the motion of binary fluid mixtures, that is fluids composed by either two phases of the same chemical species or phases of different composition. Diffuse interface models are well-known tools to describe the dynamics of complex (e.g., binary) fluids, [16]. For instance, this approach is used in [2] to describe cavitation phenomena in a flowing liquid. The model consists of the NS equation coupled with the phase-field system, [5,16,15,17]. In the isothermal compressible case, the existence of a global weak solution is proved in [13]. In the incompressible isothermal case, neglecting chemical reactions and other forces, the model reduces to an evolution system which governs the fluid velocity v and the order parameter ϕ. This system can be written as a NS equation coupled with a convective Allen–Cahn equation, [16]. The associated initial and boundary value problem was studied in [16] in which the authors proved that the system generated a strongly continuous semigroup on a suitable phase space which possesses a global attractor. They also established the existence of an exponential attractor. This entails that the global attractor has a finite fractal dimension, which is estimated in [16] in terms of some model parameters. The dynamic of simple single-phase fluids has been widely investigated although some important issues remain unresolved, [32]. In the case of binary fluids, the analysis is even more complicate and the mathematical studied is still at it infancy as noted in [16]. As noted in [15], the mathematical analysis of binary fluid flows is far from being well understood. For instance, the spinodal decomposition under shear consists of a two-stage evolution of a homogeneous initial mixture: a phase separation stage in which some macroscopic patterns appear, then a shear stage in which these patters organize themselves into parallel layers (see, e.g. [26] for experimental snapshots). This model has to take into account the chemical interactions between the two phases at the interface, achieved using a Cahn–Hilliard approach, as well as the hydrodynamic properties of the mixture (e.g., in the shear case), for which a Navier–Stokes equations with surface tension terms acting at the interface are needed. When the two fluids have the same constant density, the temperature differences are negligible and the diffuse interface between the two phases has a small but non-zero thickness, a well-known model is the so-called "Model H" (cf. [19]). This is a system of equations where an incompressible Navier–Stokes equation for the (mean) velocity v is coupled with a convective Cahn–Hilliard equation for the order parameter ϕ, which represents the relative concentration of one of the fluids.

Many challenges in the mathematical and numerical analysis of the Allen–Cahn–Navier–Stokes equations (AC-NSE) or the Cahn–Hilliard–Navier–Stokes equations CH-NSE) are related to the fact that the full mathematical theory for the 3D Navier–Stokes equation (NSE) is still lacking at present. Since the uniqueness theorem for the global weak solutions (or the global existence of strong solutions) of the initial-value problem of the 3D Navier–Stokes system is not proved yet, the known theory of global attractors of infinite dimensional dynamical systems is not applicable to the 3D Navier–Stokes system. Using regular approximation equations to study the classical 3D Navier–Stokes systems has become an effective tool both from the numerical and the theoretical point of views. As noted in [34], it was demonstrated analytically and numerically in many works that the LANS-α model gives a good approximation in the study of many problems related to turbulence flows. In particular, it was found that the explicit steady analytical solution of the LANS-α model compare successfully with empirical and numerical experiment data for a wide range of Reynolds numbers in turbulent channel and pipe flows, [34]. Let us recall that the inviscid 3D LANS-α equations was first proposed in [21,20]. As described in [24], the 3D LANS-α equations are a systems of partial differential equations for the mean velocity in which a nonlinear dispersive mechanism filters the small scales. As such, the 3D LANS-α equations serve as an appropriate model for turbulent flows and a suitable approximation of the 3D NS as documented in [7,9,8,10].

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