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Principal eigenvalue of mixed problem for the fractional Laplacian: Moving the boundary conditions ☆

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Abstract

We analyze the behavior of the eigenvalues of the following nonlocal mixed problem

$$\begin{cases} (-\Delta)^s u = \lambda_1(D)\, u & \text{in } \Omega, \\ u = 0 & \text{in } D, \\ \mathcal{N}_s u = 0 & \text{in } N. \end{cases}$$

Our goal is to construct different sequences of problems by modifying the configuration of the sets D and N, and to provide sufficient and necessary conditions on the size and the location of these sets in order to obtain sequences of eigenvalues that in the limit recover the eigenvalues of the Dirichlet or Neumann problem. We will see that the nonlocality plays a crucial role here, since the sets D and N can have infinite measure, a phenomenon that does not appear in the local case (see for example [7,8,6]).

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1. Introduction

In the papers [7,8], J. Denzler considers the following mixed Dirichlet–Neumann eigenvalue problem

$$\begin{cases} -\Delta u = \lambda_1(D)u & \text{in } \Omega, \\ u = 0 & \text{in } D, \\ \dfrac{\partial u}{\partial n} = 0 & \text{in } N. \end{cases} \tag{1.1}$$

Here Ω is a Lipschitz bounded domain in $\mathbb{R}^N$, D, N are submanifolds of $\partial\Omega$ such that

$$\bar{D} \cup \bar{N} = \partial\Omega \quad \text{and} \quad D \cap N = \emptyset, \tag{1.2}$$

and $\lambda_1(D)$ is the first eigenvalue; that is, if $H^1_D(\Omega) = \{u \in H^1(\Omega) \,|\, u = 0 \text{ on } D \subset \partial\Omega\}$, then

$$\lambda_1(D) := \inf_{u \in H^1_D(\Omega),\ u \not\equiv 0} \frac{\displaystyle\int_\Omega |\nabla u|^2\,dx}{\displaystyle\int_\Omega |u|^2\,dx}.$$

In his papers, Denzler studies the behavior of this eigenvalue according to the configuration of the sets with Dirichlet (or conversely, with Neumann) condition. More precisely, he constructs different examples describing the way in which the geometric arrangement of the Dirichlet part (for a fixed measure) affects the size of the corresponding eigenvalue. Indeed, he shows the following property.

Theorem 1.1 *([8], Theorem 5 and Theorem 6). Given* $0 < \alpha \leqslant |\partial\Omega|$, *and*

$$\mu := \inf\{\lambda_1(D) : \ |D| = \alpha\} > 0,$$

there exists a configuration set $D_0 \subset \partial\Omega$ *with* $|D_0| = \alpha$ *such that*

$$\lambda_1(D_0) = \mu.$$

That is, for an admissible value α and all the possible configurations in problem (1.1) of the boundary conditions with the Dirichlet part given by a set of measure equal to α, the infimum of the corresponding eigenvalues is positive and can be attained. In other words, there exists a configuration whose associated eigenvalue is this infimum. In a franctional setting a related result can be seen in [11].