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YJDEQ:9248

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J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Fast propagation for reaction–diffusion cooperative systems

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Received 7 September 2017; revised 2 March 2018

Abstract

This paper deals with the spatial propagation for reaction–diffusion cooperative systems. It is well-known that the solution of a reaction–diffusion equation with monostable nonlinearity spreads at a finite speed when the initial condition decays to zero exponentially or faster, and propagates fast when the initial condition decays to zero more slowly than any exponentially decaying function. However, in reaction–diffusion cooperative systems, a new possibility happens in which one species propagates fast although its initial condition decays exponentially or faster. The fundamental reason is that the growth sources of one species come from the other species. Simply speaking, we find a new interesting phenomenon that the spatial propagation of one species is accelerated by the other species. This is a unique phenomenon in reaction–diffusion systems. We present a framework of fast propagation for reaction–diffusion cooperative systems.

MSC: 35C07; 35K57; 92D25

Keywords: Spatial propagation; Reaction–diffusion cooperative systems; Accelerating fronts

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https://doi.org/10.1016/j.jde.2018.03.004
0022-0396/

1. Introduction

1.1. Spatial propagation of reaction–diffusion equations

One interesting problem in the study of parabolic partial differential equations is the spatial propagation of the following reaction–diffusion equation

$$\begin{cases} u_t(t,x) = u_{xx}(t,x) + f(u(t,x)), \ t > 0, \ x \in \mathbb{R}, \\ u(0,x) = u_0(x), \ x \in \mathbb{R}, \end{cases} \tag{1.1}$$

with Fisher-KPP nonlinearity $f(u)$ and continuous front-like initial condition $u_0(x)$, in the sense that

$$f(0) = f(1) = 0, \ 0 < f(u) \leqslant f'(0)u \text{ for } u \in (0,1),$$

$$\liminf_{x\to-\infty} u_0(x) > 0, \ \lim_{x\to+\infty} u_0(x) = 0 \text{ and } 0 \leqslant u_0(x) < 1 \text{ for } x \in \mathbb{R}.$$

For many front-like initial conditions, the spatial propagation of a solution at large time is related to the traveling wave solution $u(t,x) = \phi_c(x - ct)$ satisfying

$$\begin{cases} \phi_c''(\xi) + c\phi_c'(\xi) + f(\phi_c(\xi)) = 0, \\ \phi_c(-\infty) = 1, \ \phi_c(+\infty) = 0, \end{cases} \tag{1.2}$$

where $\xi = x - ct$. It is well-known that equation (1.2) with Fisher-KPP nonlinearity admits a solution $\phi_c(\xi)$ for speed $c \geqslant c^* = 2\sqrt{f'(0)}$, and does not have any solution when $0 < c < c^*$. Therefore, the constant c^* is called the *minimal speed* (for the existence of traveling wave solutions).

It is widely shown that the decay behavior of $u_0(x)$ to zero determines the spatial propagation of $u(t,x)$ at large time. First, if

$$u_0(x) = 0 \text{ in } [x_0, +\infty) \tag{1.3}$$

for some constant $x_0 \in \mathbb{R}$, then $u(t,x)$ approaches a traveling wave solution as follows

$$\lim_{t\to+\infty} u(t, x + m(t)) = \phi_{c^*}(x), \tag{1.4}$$

where $m(t) \sim c^*t + O(\ln t)$ as $t \to +\infty$. If $u_0(x)$ satisfies that

$$u_0(x) \sim O(e^{-\lambda x}) \text{ as } x \to +\infty, \tag{1.5}$$

where λ is a positive constant, then (1.4) holds in case of $\lambda \geqslant \sqrt{f'(0)}$, and when $\lambda < \sqrt{f'(0)}$, the following approach holds

$$\lim_{t\to+\infty} u(t, x + m(t)) = \phi_c(x), \tag{1.6}$$

Please cite this article in press as: W.-B. Xu et al., Fast propagation for reaction–diffusion cooperative systems, J. Differential Equations (2018), https://doi.org/10.1016/j.jde.2018.03.004