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Competition in periodic media: II – Segregative limit of pulsating fronts and "Unity is not Strength"-type result ☆

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Abstract

This paper is concerned with the limit, as the interspecific competition rate goes to infinity, of pulsating front solutions in space-periodic media for a bistable two-species competition–diffusion Lotka–Volterra system. We distinguish two important cases: null asymptotic speed and non-null asymptotic speed. In the former case, we show the existence of a segregated stationary equilibrium. In the latter case, we are able to uniquely characterize the segregated pulsating front, and thus full convergence is proved. The segregated pulsating front solves an interesting free boundary problem. We also investigate the sign of the speed as a function of the parameters of the competitive system. We are able to determine it in full generality, with explicit conditions depending on the various parameters of the problem. In particular, if one species is sufficiently more motile or competitive than the other, then it is the invader. This is an extension of our previous work in space-homogeneous media.

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Introduction

This is the second part of a sequel to our previous article [24]. In the prequel, we studied the sign of the speed of bistable traveling wave solutions of the following competition–diffusion problem:

$$\begin{cases} \partial_t u_1 - \partial_{xx} u_1 = u_1 (1 - u_1) - k u_1 u_2 & \text{in } (0, +\infty) \times \mathbb{R} \\ \partial_t u_2 - d \partial_{xx} u_2 = r u_2 (1 - u_2) - \alpha k u_1 u_2 & \text{in } (0, +\infty) \times \mathbb{R}. \end{cases}$$

We proved that, as $k \to +\infty$, the speed of the traveling wave connecting $(1, 0)$ to $(0, 1)$ converges to a limit which has exactly the sign of $\alpha^2 - rd$. In particular, if $\alpha = r = 1$ and if k is large enough, the more motile species is the invader: this is what we called the "Unity is not strength" result.