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# Oseen resolvent estimates with small resolvent parameter

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**Abstract**

We consider the Oseen system with resolvent term in an exterior domain in $\mathbb{R}^3$, supplemented by homogeneous Dirichlet boundary conditions. Under the assumption that the resolvent parameter $\lambda$ is close to zero and $\Re\lambda \geq 0$, $\lambda \neq 0$, we estimate the $L^p$-norm of the velocity against the $L^p$-norm of the right-hand side, times a factor $C\,|\lambda|^{-2}$, with $C > 0$ independent of $\lambda$. Such an estimate cannot hold for this range of $\lambda$ if $|\lambda|^{-2}$ is replaced by $|\lambda|^{-\kappa}$ with $\kappa < 3/2$, and there are indications that $\kappa \in [3/2, 2)$ cannot be admitted either. We present various other $L^p$-estimates of Oseen resolvent flows for the same range of $\lambda$. Our article is complementary to the work by T. Kobayashi and Y. Shibata (1998) [20], where Oseen resolvent estimates are derived under the assumption that $|\lambda| \geq c_0$, for some arbitrary but fixed $c_0 > 0$, with the constant in the resolvent estimate depending on $c_0$.
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## 1. Introduction

An incompressible viscous flow around a rigid body moving steadily and without rotation is usually described by the Navier–Stokes system with an Oseen term,

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$$\partial_t u - \Delta_x u + \tau\, \partial_{x_1} u + \tau\, (u \cdot \nabla_x) u + \nabla_x \pi = f, \quad \operatorname{div}_x u = 0 \quad \text{in } \Omega \times (0, \infty), \tag{1.1}$$

where $\Omega$ is an "exterior domain" in $\mathbb{R}^3$, that is, an open set in $\mathbb{R}^3$ with bounded complement. The set $\mathbb{R}^3 \backslash \overline{\Omega}$ represents the rigid body, and $\tau \in (0, \infty)$ is the Reynolds number. The unknowns of this system are the velocity field $u$ and the pressure field $\pi$ of the fluid. The velocity in question is the "velocity above ground". This means that the fluid particle located at point $x = (x_1, x_2, x_3)$ at time $t$ moves with velocity $u(x,t)$ with respect to a fixed point in space, but $x_1$, $x_2$, $x_3$ are the coordinates of that particle with respect to a reference system which adheres to the rigid body, and so is not attached to the fixed point in space. This type of velocity has two advantages from a mathematical point of view. Firstly, the rigid body is represented by a set that does not depend on time, and secondly, the velocity of the fluid vanishes at infinity in the sense that $u(x,t) \to 0$ for $|x| \to \infty$, a condition that may be expressed in weak form by the relation $u(\,\cdot\,, t) \in L^r(\Omega)^3$ for $t \in (0, \infty)$, where $r$ is some number in $[1, \infty)$. But on the other hand an additional term – the Oseen term $\tau\, \partial_{x_1} u$ – arises in system (1.1), complicating some aspects of the mathematical theory related to that system. In the work at hand, we will deal with such an aspect. In fact, we consider the Oseen resolvent system

$$-\Delta u + \tau\, \partial_1 u + \lambda\, u + \nabla \pi = f, \;\; \operatorname{div} u = 0 \quad \text{in } \Omega \tag{1.2}$$

as well as the stationary Oseen system

$$-\Delta u + \tau\, \partial_1 u + \nabla \pi = f, \;\; \operatorname{div} u = 0 \quad \text{in } \Omega, \tag{1.3}$$

which may be considered as a special case of (1.2) ($\lambda = 0$). In order to explain what we want to show with respect to (1.2) and (1.3), let us first recall some facts about the Stokes resolvent system

$$-\Delta u + \lambda\, u + \nabla \pi = f, \;\; \operatorname{div} u = 0 \quad \text{in } \Omega, \tag{1.4}$$

under Dirichlet boundary conditions

$$u | \partial\Omega = 0. \tag{1.5}$$

If $S \in (0, \infty)$ with $\mathbb{R}^3 \backslash \Omega \subset B_S$, $p \in (1, \infty)$ and $\vartheta \in (\pi/2, \pi)$, then for any $\lambda \in \mathbb{C} \backslash \{0\}$ with $|\arg \lambda| \le \vartheta$, there is a unique pair of functions $(u, \pi)$ such that $u \in W^{2,p}(\Omega)^3 \cap W_0^{1,p}(\Omega)^3$, $\pi \in W^{1,p}_{loc}(\Omega)$, $\nabla \pi \in L^p(\Omega)^3$ and $\int_{B_S \cap \Omega} \pi \, dx = 0$, and such that (1.4) is satisfied. In addition, there is a constant $C > 0$, only depending on $\Omega$, $p$ and $\vartheta$, such that

$$\|u\|_p \le C\, |\lambda|^{-1}\, \|f\|_p \quad \text{for } \lambda,\ f \text{ and } u \text{ as above;} \tag{1.6}$$

see [19], [3], [5], [6], [7], [15], [27]. Inequality (1.6) is a basic tool in the mathematical study of the Navier–Stokes system

$$\partial_t u - \Delta_x u + \tau\, (u \cdot \nabla_x) u + \nabla_x \pi = f, \quad \operatorname{div}_x u = 0 \quad \text{in } \Omega \times (0, \infty),$$

and is applied directly or indirectly in a very large number of articles dealing with this system, usually in the context of the theory of analytic semigroups.

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