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# Ground state solution for a class of indefinite variational problems with critical growth

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**Abstract**

In this paper we study the existence of ground state solution for an indefinite variational problem of the type

$$\begin{cases} -\Delta u + (V(x) - W(x))u = f(x,u) \quad \text{in} \quad \mathbb{R}^N, \\ u \in H^1(\mathbb{R}^N), \end{cases} \tag{$P$}$$

where $N \geq 2$, $V, W : \mathbb{R}^N \to \mathbb{R}$ and $f : \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}$ are continuous functions verifying some technical conditions and $f$ possesses a critical growth. Here, we will consider the case where the problem is asymptotically periodic, that is, $V$ is $\mathbb{Z}^N$-periodic, $W$ goes to 0 at infinity and $f$ is asymptotically periodic.


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*Keywords:* Critical growth; Variational methods; Elliptic equations; Indefinite strongly functional

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## 1. Introduction

In this paper we study the existence of ground state solution for an indefinite variational problem of the type

$$\begin{cases} -\Delta u + (V(x) - W(x))u = f(x,u), \ \text{in } \mathbb{R}^N, \\ u \in H^1(\mathbb{R}^N), \end{cases} \tag{$P$}$$

where $N \geq 2$, $V, W : \mathbb{R}^N \to \mathbb{R}$ are continuous functions verifying some technical conditions and $f$ has a critical growth. Here, we will consider the case where the problem is asymptotically periodic, that is, $V$ is $\mathbb{Z}^N$-periodic, $W$ goes to 0 at infinity and $f$ is asymptotically periodic.

In [13], Kryszewski and Szulkin have studied the existence of ground state solution for an indefinite variational problem of the type

$$\begin{cases} -\Delta u + V(x)u = f(x,u), \quad \text{in} \quad \mathbb{R}^N, \\ u \in H^1(\mathbb{R}^N), \end{cases} \tag{$P_1$}$$

where $V : \mathbb{R}^N \to \mathbb{R}$ is a $\mathbb{Z}^N$-periodic continuous function such that

$$0 \notin \sigma(-\Delta + V), \text{ the spectrum of } -\Delta + V. \tag{$V_1$}$$

Related to the function $f : \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}$, they assumed that $f$ is continuous, $\mathbb{Z}^N$-periodic in $x$ with

$$|f(x,t)| \leq c(|t|^{q-1} + |t|^{p-1}), \quad \forall t \in \mathbb{R} \quad \text{and} \quad x \in \mathbb{R}^N \tag{$h_1$}$$

and

$$0 < \alpha F(x,t) \leq t f(x,t) \quad \forall t \in \mathbb{R}, \quad F(x,t) = \int_0^t f(x,s)\,ds \tag{$h_2$}$$

for some $c > 0$, $\alpha > 2$ and $2 < q < p < 2^*$ where $2^* = \frac{2N}{N-2}$ if $N \geq 3$ and $2^* = +\infty$ if $N = 2$. The above hypotheses guarantee that the energy functional associated with ($P_1$) given by

$$J(u) = \frac{1}{2}\int_{\mathbb{R}^N} (|\nabla u|^2 + V(x)|u|^2\,dx) - \int_{\mathbb{R}^N} F(x,u)\,dx, \ u \in H^1(\mathbb{R}^N),$$

is well defined and belongs to $C^1(H^1(\mathbb{R}^N), \mathbb{R})$. By ($V_1$), there is an equivalent inner product $\langle\ ,\ \rangle$ in $H^1(\mathbb{R}^N)$ such that

$$J(u) = \frac{1}{2}\|u^+\|^2 - \frac{1}{2}\|u^-\|^2 - \int_{\mathbb{R}^N} F(x,u)\,dx,$$

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