ARTICLE IN PRESS YJDEQ:9214

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J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Periodic perturbations with rotational symmetry of planar systems driven by a central force

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Received 12 December 2017; revised 6 February 2018

Abstract

We consider periodic perturbations of a central force field having a rotational symmetry, and prove the existence of nearly circular periodic orbits. We thus generalize, in the planar case, some previous bifurcation results obtained by Ambrosetti and Coti Zelati in [1]. Our results apply, in particular, to the classical Kepler problem.

MSC: 34C25

Keywords: Kepler problem; Periodic solutions; Rotational symmetric systems

1. Introduction

We study the planar system

$$\ddot{x} + g(|x|)\,x = \varepsilon \nabla_x V(t, x)\,, \qquad (S_\varepsilon)$$

where $g :]0, +\infty[\to \mathbb{R}$ is a continuously differentiable function and $V : \mathbb{R} \times (\mathbb{R}^2 \setminus \{0\}) \to \mathbb{R}$ is continuous, T-periodic in its first variable, and twice continuously differentiable in its second variable. The real number ε will be considered as a small parameter.

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https://doi.org/10.1016/j.jde.2018.02.009
0022-0396/

A particularly important special case of (S_ε) is illustrated by the perturbed Kepler problem

$$\ddot{x} + \frac{c}{|x|^3}\,x = \varepsilon \nabla_x V(t,x)\,, \tag{K_ε}$$

where c is a positive constant. This kind of systems has been studied by plenty of authors, mainly with the aim of finding periodic solutions: see, e.g., [1,2,4–6,8–14,17,19]. Let us describe two of these results in detail.

In 1989, Ambrosetti and Coti Zelati [1] considered system (K_ε) in any dimension, and proved that periodic solutions of period $2T$ exist assuming the perturbing potential to be *even* in x, i.e., that

$$V(t,x) = V(t,-x)\,, \qquad \text{for every } t, x\,.$$

Their method of proof is variational, and provides a bifurcation result from a circular solution $x_*(t)$ of (K_0) having minimal period $\tau_* = 2T/n$, with n odd. (By *circular solution* we mean a solution whose orbit is a circle centered at the origin.) Indeed, they proved that at least three such solutions $x(t)$ exist, satisfying the symmetry property

$$x(t+T) = -x(t)\,, \qquad \text{for every } t\,.$$

Recently, the authors of the present paper considered in [10] the case when the perturbing force is *radially symmetric*, i.e., when $\nabla_x V(t,x) = p(t,|x|)\,x$, for some scalar function $p : \mathbb{R}\times\,]0,+\infty[\,\to \mathbb{R}$. It was proved that, if $x_*(t)$ is a circular solution having minimal period τ_*, and T/τ_* is a rational number n/m which is not an integer, then near this circular solution there are mT-periodic solutions $x(t)$ of system (K_ε) making exactly n rotations around the origin in their period time, provided that $|\varepsilon|$ is small enough. Moreover, $t \mapsto |x(t)|$ is T-periodic, and $t \mapsto \mathrm{Rot}(x,[t,t+T])$ is constant.

Let us recall here that, when $\gamma : [\tau_1,\tau_2] \to \mathbb{R}^2$ is a curve such that $\gamma(t) \neq 0$ for every t, writing $\gamma(t) = \rho(t)(\cos\theta(t), \sin\theta(t))$, all functions being continuous, the *rotation number* around the origin is defined as

$$\mathrm{Rot}\,(\gamma;[\tau_1,\tau_2]) = \frac{\theta(\tau_2)-\theta(\tau_1)}{2\pi}\,.$$

In this paper we assume that the perturbing force has a *rotational symmetry*, i.e., there is a rotation around the origin $\mathcal{R}$, of angle

$$\vartheta = \frac{2\pi}{m}\,,$$

with m being a positive integer, such that

$$\nabla_x V(t,\mathcal{R}x) = \mathcal{R}\nabla_x V(t,x)\,, \qquad \text{for every } t, x\,. \tag{1}$$

Notice that, if $m = 1$, the above assumption will be trivially satisfied.

Please cite this article in press as: A. Fonda, A.C. Gallo, Periodic perturbations with rotational symmetry of planar systems driven by a central force, J. Differential Equations (2018), https://doi.org/10.1016/j.jde.2018.02.009