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Measure-valued solutions to nonlocal transport equations on networks

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Abstract

Aiming to describe traffic flow on road networks with long-range driver interactions, we study a nonlinear transport equation defined on an oriented network where the velocity field depends not only on the state variable but also on the distribution of the population. We prove existence, uniqueness and continuous dependence results of the solution intended in a suitable measure-theoretic sense. We also provide a representation formula in terms of the push-forward of the initial and boundary data along the network and discuss an explicit example of nonlocal velocity field fitting our framework.

MSC: 35R02; 35Q35; 28A50

Keywords: Network; Transport equation; Measure-valued solution; Transmission conditions

1. Introduction

In recent times there has been a considerable amount of literature devoted to the study of evolution equations in measures spaces. Indeed the measure-valued approach presents, with re-

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spect to other approaches based on classical and weak solutions, some significant advantages: the population is represented by a probability distribution, providing a unified framework for both discrete and continuous models; short and long range interaction mechanisms are efficiently described by taking a velocity field depending on local terms, determined by the geometry of the space, and nonlocal terms, regulated by the position of the other individuals, hence on the whole measure; aggregation phenomena that in a classical setting lead to blow-up of the solution are plainly taken into account by the measure setting. The by now classical reference for evolution equation in measure spaces is the book [1], while we refer to [3,7,10,16,17] for various applications to the study of complex phenomena. However most of the literature about measure-valued equations considers these problems in the full space, because their study in bounded domains poses additional difficulties due in particular to the interpretation of the boundary conditions. For the specific case of a bounded interval, an interpretation of the boundary condition in a measure sense has been pursued in [8,9], while in [15] a measure-valued transport equation on a sequence of intervals with a transmission condition at intersection points is considered.

Motivated by pedestrian and vehicular traffic modelling in urban areas, several models have been proposed for traffic flow on road networks, see [2,11,12] and references therein. Most of these models are based on a fluid-dynamical approach and take into account only local interactions among drivers, the main purpose being to find appropriate rules at the junctions, namely the vertices of the network, to optimize the traffic flow.

In order to extend the measure-valued approach to networks, in [4] it was studied the linear transport equation

$$\partial_t m + \partial_x(v(x)m) = 0 \qquad \text{in } \Gamma \times [0, T] \tag{1.1}$$

where Γ is an oriented network. Existence, uniqueness and continuous dependence results for the measure-valued solution to (1.1) were provided, along with a local representation formula on each arc. Even if this simplified model already presents some interesting peculiarities and difficulties due to the presence of the junction conditions, nonlocal driver interactions were not included in the model since v was assumed to depend only on the space variable.

The aim of this paper is to study measure-valued nonlinear transport equations on networks where the velocity field depends on the measure itself. More precisely, we consider the nonlinear transport equation

$$\partial_t m + \partial_x(v[m_t]m) = 0, \qquad \text{in } \Gamma \times [0, T] \tag{1.2}$$

where the velocity v still depends on the x-variable, but also on the vehicle distribution m_t at time t. To explain the main difference between (1.1) and (1.2) we observe that (1.1) is formally equivalent to a system of equations, one for each arc, coupled via the transmission conditions at the vertices. Instead, in (1.2) the evolution equation in each arc does not only depend on the distribution of the vehicles flowing into the arc from the junction but also on the (global) distribution m_t at time t on Γ.

To show the well posedness of (1.2) we approximate the nonlinear transport equation by a sequence of linear problems obtained via a semi-discrete-in-time approximation of (1.2). We define a partition of the time interval $[0, T]$ in a family of subinterval $[t_k, t_k + \Delta t]$ and on each of these intervals we solve the linear problem (1.1) with the nonlinear velocity $v[m_t]$ replaced by the linear one $v[m_{t_k}]$. In such a way we obtain a sequence of measure $\{m^{\Delta t}\}$ defined on $[0, T]$. Using the results on the linear problem, we prove that for $\Delta t \to 0^+$ the sequence $\{m^{\Delta t}\}$

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