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Global bifurcation of solutions of the mean curvature spacelike equation in certain Friedmann–Lemaître–Robertson–Walker spacetimes ☆

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Abstract

We study the existence of spacelike graphs for the prescribed mean curvature equation in the Friedmann–Lemaître–Robertson–Walker (FLRW) spacetime. By using a conformal change of variable, this problem is translated into an equivalent problem in the Lorentz–Minkowski spacetime. Then, by using Rabinowitz's global bifurcation method, we obtain the existence and multiplicity of positive solutions for this equation with 0-Dirichlet boundary condition on a ball. Moreover, the global structure of the positive solution set is studied.

MSC: 35B32; 53A10; 35B40; 53B30; 53C50

Keywords: Bifurcation; Mean curvature spacelike equation; Positive solution; Friedmann–Lemaître–Robertson–Walker spacetime

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1. Introduction and main results

Let $I \subseteq \mathbb{R}$ be an open interval endowed with the (negative definite) metric $-dt^2$. Denote by $\mathcal{M}$ the $(N+1)$-dimensional product manifold $I \times \mathbb{R}^N$ with $N \geq 1$ endowed the Lorentzian metric

$$g = -dt^2 + f^2(t)dx^2, \tag{1.1}$$

where $f \in C^\infty(I)$, $f > 0$, is called the scale factor or warping function. Clearly, $\mathcal{M}$ is a Lorentzian warped product, in the sense of [21], with base $\left(I, -dt^2\right)$, fiber $\left(\mathbb{R}^N, dx^2\right)$ and warping function f. This type of spacetimes plays a central role in General Relativity. For dim $\mathcal{M} = 4$, t may be interpreted as the relative time of a family of privileged observers, the so-called comoving observers, and for them the quantity $f(t)$ is the radius of their spatial universe at time t. Then, the positive (resp. negative) sign of $f'(t)$ indicates that these observers perceive expansion (resp. contraction) at a given time t. Moreover, for warping functions close to 1, the corresponding spacetime $\mathcal{M}$ may be thought as a deformation of the Lorentz–Minkowski spacetime, so these spacetimes are good candidates to explore stability of physical properties expressed for an empty universe in terms of the Lorentz–Minkowski spacetime. In this paper, we will refer $\mathcal{M}$ as a (flat fiber) Friedmann–Lemaître–Robertson–Walker (FLRW) spacetime. More generally, if the fiber of $\mathcal{M}$ is changed to an N-dimensional Riemannian manifold of constant sectional curvature we arrive to the notion of (general) FLRW spacetime.

In the four dimensional case, FLRW spacetimes have been useful to obtain exact solutions of Einstein's field equations of General Relativity because they describe spatially homogeneous and isotropic (expanding or contracting) universes. These geometric properties are in complete agreement with the experience and, therefore, these models have been useful to describe the large scale of the universe from the point of view of the relativistic cosmology. The pioneering and important results obtained by the use of FLRW models were first derived by Friedmann in 1922 and 1924 [14,15]. In 1927, Lemaître [18] arrived independently at similar results as those of Friedmann. Robertson and Walker explored later the problem further during the 1930s [25–27,30]. In particular, Robertson rigorously proved that a spatially homogeneous and isotropic spacetime must be locally isometric to an FLRW spacetime in 1935. For more details of FLRW spacetimes, see [21, Chapter 12] or the monograph of Choquet-Bruhat [9] and the references therein.

Given $f \in C^\infty(I)$, $f > 0$, for each $u \in C^\infty(\Omega)$, where Ω is a domain of $\mathbb{R}^N$, such that $u(\Omega) \subset I$ we can consider its graph $M = \left\{(u(x), x) : x \in \Omega\right\}$ in the FLRW spacetime $\mathcal{M}$. The graph inherits a metric from (1.1), given by

$$-du^2 + f^2(u)dx^2 \tag{1.2}$$

on Ω, which is positive definite if and only if u satisfies

$$|\nabla u| < f(u) \tag{1.3}$$

everywhere on Ω, where ∇u is the gradient of u in $\mathbb{R}^N$ and $|\nabla u|$ its length. When the metric (1.2) is Riemannian, the graph M is called spacelike. In this case, the pointing future unitary normal vector field on M is given by