ARTICLE IN PRESS YJDEQ:9218

Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Regularity for degenerate evolution equations with strong absorption

João Vitor da Silva [a], Pablo Ochoa [b], Analia Silva [c]

[a] *Universidad de Buenos Aires, Departamento de Matemática – FCEyN – Ciudad Universitaria, Pabellón I (1428), Buenos Aires, Argentina*

[b] *Universidad Nacional de Cuyo, and CONICET, Mendoza 5500, Argentina*

[c] *Instituto de Matemática Aplicada San Luis-IMASL, Universidad Nacional de San Luis and CONICET, Ejercito de los Andes 950, D5700HHW San Luis, Argentina*

Received 1 July 2017; revised 19 January 2018

Abstract

In this manuscript, we study geometric regularity estimates for degenerate parabolic equations of p-Laplacian type ($2 \le p < \infty$) under a strong absorption condition:

$$\Delta_p u - \frac{\partial u}{\partial t} = \lambda_0 u_+^q \quad \text{in} \quad \Omega_T := \Omega \times (0, T),$$

where $0 \le q < 1$. This model is interesting because it yields the formation of dead-core sets, i.e., regions where non-negative solutions vanish identically. We shall prove sharp and improved parabolic C^α regularity estimates along the set $\mathfrak{F}_0(u, \Omega_T) = \partial\{u > 0\} \cap \Omega_T$ (the free boundary), where $\alpha = \frac{p}{p-1-q} \ge 1 + \frac{1}{p-1}$. Some weak geometric and measure theoretical properties as non-degeneracy, positive density, porosity and finite speed of propagation are proved. As an application, we prove a Liouville-type result for entire solutions. A specific analysis for Blow-up type solutions will be done as well. The results are new even for dead-core problems driven by the heat operator.

MSC: 35B53; 35B65; 35J60; 35K55; 35K65

E-mail addresses: jdasilva@dm.uba.ar (J.V. da Silva), ochopablo@gmail.com (P. Ochoa), asilva@dm.uba.ar (A. Silva).

https://doi.org/10.1016/j.jde.2018.02.013
0022-0396/© 2018 Published by Elsevier Inc.

Please cite this article in press as: J.V. da Silva et al., Regularity for degenerate evolution equations with strong absorption, J. Differential Equations (2018), https://doi.org/10.1016/j.jde.2018.02.013

Keywords: p-Laplacian type operators; Dead-core problems; Sharp and improved intrinsic regularity; Liouville type results

1. Introduction

Throughout the last 40 years a wide class of parabolic equations has been used to model problems coming from chemical reactions, physical-mathematical phenomena, biological processes, population dynamics, among others. Some of the main topics approached are reaction–diffusion processes with one-phase transition. Thus, the existence of non-negative solutions plays an essential role in these studies (cf. [7], [28] and references therein). An enlightening prototype is the following model of an isothermal catalytic reaction–diffusion process:

$$\begin{cases} \Delta u - \frac{\partial u}{\partial t} = f(u) & \text{in } \Omega_T \\ u(x,t) = g(x,t) & \text{on } \partial\Omega \times (0,T) \\ u(x,0) = u_0(x) & \text{in } \overline{\Omega}, \end{cases} \tag{1.1}$$

where the boundary data fulfils

$$0 < u_0 \in C^0(\overline{\Omega}), \quad g(x,t) = \mathrm{j} > 0 \quad \text{and} \quad u(x,0) = \mathrm{j} \ \forall\, x \in \partial\Omega.$$

Here, u represents the concentration of a reactant evolving in time, where $\Omega_T := \Omega \times (0,T)$, $\Omega \subset \mathbb{R}^N$ is a regular and bounded domain and f is a non-linear convection term fulfilling $f(s) > 0$ if $s > 0$ and $f(0) = 0$. Moreover, the boundary conditions mean that the reactant is injected with a constant isothermal flux on the boundary. Recall that in evolution problems when f is locally Lipschitz, it follows from the Maximum Principle that non-negative solutions must be strictly positive. However when f is not Lipschitz (or not decaying sufficiently fast) at the origin, then non-negative solutions may exhibit a plateau region, which is known in the literature as *Dead-Core* set, i.e., a region of positive measure where non-negative solutions vanish identically.

Advances in connection with existence theory of dead-core solutions, formation of dead-core regions and decay estimates at infinity were obtained in [7], [21] and [27]. Other properties of solutions such as growth of interfaces, shrink and estimates of/on the support and finite extinction in reaction–diffusion problems together with other qualitative properties may be found in Díaz et al.'s fundamental articles [1], [3], [16], [17], the Antontsev et al.'s classical book [4], the survey [15] and references therein. However, the lack of quantitative properties for p-parabolic dead-core problems constitutes our starting point in this research. In particular, we shall be interested in the derivation of quantitative results for the following class of parabolic dead-core problems of p-Laplacian type:

$$\Delta_p u - \frac{\partial u}{\partial t} = \lambda_0(x,t).u_+^q(x,t) \quad \text{in} \quad \Omega_T, \tag{1.2}$$

with suitable boundary data, where $u_+ = \max\{u, 0\}$, $0 \le q < 1$, $p \ge 2$, $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain and λ_0 (the *Thiele modulus*) is bounded away from zero and infinity. We refer the reader to Section 2 for notation and definitions.

Please cite this article in press as: J.V. da Silva et al., Regularity for degenerate evolution equations with strong absorption, J. Differential Equations (2018), https://doi.org/10.1016/j.jde.2018.02.013