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Necessary optimality conditions for infinite dimensional state constrained control problems

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Abstract

This paper is concerned with first order necessary optimality conditions for state constrained control problems in separable Banach spaces. Assuming inward pointing conditions on the constraint, we give a simple proof of Pontryagin maximum principle, relying on infinite dimensional neighboring feasible trajectories theorems proved in [20]. Further, we provide sufficient conditions guaranteeing normality of the maximum principle. We work in the abstract semigroup setting, but nevertheless we apply our results to several concrete models involving controlled PDEs. Pointwise state constraints (as positivity of the solutions) are allowed.

MSC: 49K20; 49K27; 49K30; 35Q93

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1. Introduction

The maximum principle for optimal control problems can be considered as a milestone in the theory of control. Due to its importance, an extensive literature has been devoted to this

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subject, both in finite and in infinite dimensions. The main interest of the infinite dimensional setting is due to the fact that many physical models can be formulated in this framework, as for instance heat conduction, reaction–diffusion processes, properties of elastic materials, to mention only a few of them. To optimize a measure of best performance is indeed a natural need in concrete problems. For this reason optimal control governed by PDEs is a very active field of research, see e.g. the classical books, [6,8,9,17,24,25,35], containing also rich bibliographies. In the literature two strategies can be found to deal with such an interesting topic: the abstract semigroup approach, and a direct one relying on PDEs methods. The advantage of the second approach is that many fine properties of the solutions, as regularity, can be used. In contrast, the first more general framework directly applies to a variety of models. Further, some of the techniques developed in finite dimensions can be adapted (modulo fine, and sometimes difficult, tuning) to this setting.

Among the two approaches we exploit here the first one. Nevertheless, we are convinced that some of the introduced here technical methods can be adapted also in the direct PDEs analysis. Previously, we have developed some tools in the abstract setting, see [19,20], suitable to deal with state constrained problems, that are of crucial importance in applied sciences. In particular, we have proved some neighboring feasible trajectory theorems allowing to estimate the distance between a given trajectory of an evolution system and its trajectories lying in the interior of the state constraint. This tool has been studied in depth in the finite dimensional setting, see e.g. [12, 31,32] to mention a few. To our knowledge in the infinite dimensional framework, neighboring feasible trajectory theorems have been proved for the first time in [20]. In the present paper, using this effective tool, we provide a direct proof of the Pontryagin maximum principle for an optimal control state constrained problem, together with its normality. Further, we apply our results to some concrete models involving controlled PDEs with state constraints and study some examples with pointwise state constraints, such as positivity of solutions (important when dealing with populations dynamics) or lower and upper pointwise bounds (important in heat equations to avoid damage in the material during heating processes).

In an infinite dimensional separable Banach space X, we consider the solutions $x : I = [0, 1] \to X$ of the control system

$$\dot{x}(t) = \mathbb{A}\, x(t) + f(t, x(t), u(t)), \qquad \text{a.e. } t \in I\,, \tag{1.1}$$

that satisfy an initial condition of the form

$$x(0) \in Q_0 \tag{1.2}$$

and the state constraint

$$x(t) \in K, \quad \forall\, t \in I\,. \tag{1.3}$$

Here, u is a measurable selection of a given measurable set valued map $U : I \rightsquigarrow Z$ with closed non-empty images, and Z is a complete separable metric space modeling the control set. The densely defined unbounded linear operator $\mathbb{A}$ is the infinitesimal generator of a strongly continuous semigroup $S(t) : X \to X$, the map $f : I \times X \times Z \to X$ is Fréchet differentiable with respect to the second variable x, Q_0 and K are closed subsets of X. The trajectories of (1.1) are understood in the mild sense (see [29]). Notice that we allow nonsmooth constraints, that are important in the applications (industrial, medical, economical...). In this paper we analyze a

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