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Exponential stability for the wave model with localized memory in a past history framework

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Abstract

In this paper we discuss the asymptotic stability as well as the well-posedness of the damped wave equation posed on a bounded domain Ω of $\mathbb{R}^n$, $n \geq 2$,

$$\rho(x)u_{tt} - \Delta u + \int_0^\infty g(s)\operatorname{div}[a(x)\nabla u(\cdot, t-s)]\,ds + b(x)u_t = 0,$$

subject to a locally distributed viscoelastic effect driven by a nonnegative function $a(x)$ and supplemented with a frictional damping $b(x) \geq 0$ acting on a region A of Ω, where $a = 0$ in A. Assuming that $\rho(x)$ is constant, considering that the well-known geometric control condition (ω, T_0) holds and supposing that the relaxation function g is bounded by a function that decays exponentially to zero, we prove that the solutions to the corresponding partial viscoelastic model decay exponentially to zero, even in the absence of the frictional dissipative effect. In addition, in some suitable cases where the material density $\rho(x)$ is not constant, it is also possible to remove the frictional damping term $b(x)u_t$, that is, the localized viscoelastic damping is strong enough to assure that the system is exponentially stable. The semi-linear case is also considered.

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1. Introduction

1.1. Description of the problem

In this article we establish the well-posedness as well as the exponential decay of solutions u of the following locally distributed viscoelastic damped wave model with past history

$$\begin{cases} \rho(x)u_{tt} - \Delta u + \displaystyle\int_0^\infty g(s)\mathrm{div}[a(x)\nabla u(\cdot, t-s)]\,ds + b(x)u_t = 0 \ \text{ in } \ \Omega \times (0,\infty), \\ u = 0 \ \text{ on } \ \partial\Omega \times \mathbb{R}, \\ u(x,s) = u^0(x,s), \ u_t(x,s) = \partial_t u^0(x,s), \quad (x,s) \in \Omega \times (-\infty, 0], \end{cases} \tag{1.1}$$

where Ω is an open bounded and connected set of $\mathbb{R}^n$, $n \geq 2$, with smooth boundary $\partial\Omega$. Here, $\rho(x) > 0$ is the density of the material and is given by a smooth function, g represents the memory kernel, $a(x) \geq 0$ is a smooth function, $b(x) \geq 0$ is a bounded function acting effectively in a region A of the domain where $a = 0$ in A and $u^0 : \Omega \times (-\infty, 0] \to \mathbb{R}$ is the prescribed past history of u. The semi-linear version of problem (1.1) will be considered in Section 4.

According to Dafermos [17] and following Giorgi, Rivera and Pata [25], let us define a new variable η corresponding to the relative displacement history,

$$\eta^t(x,s) = u(x,t) - u(x,t-s), \quad x \in \Omega, \ t \geq 0, \ s \in (0,\infty). \tag{1.2}$$

Then, proceeding formally, we obtain

$$\begin{cases} \eta_t + \eta_s = u_t \ \text{ in } \ \Omega \times (0,\infty) \times (0,\infty), \\ \eta^0(x,s) = u^0(x,0) - u^0(x,-s), \quad (x,s) \in \Omega \times (0,\infty), \\ \eta^t(x,0) := \lim\limits_{s\to 0^+} \eta^t(x,0) = 0, \quad (x,t) \in \Omega \times [0,\infty). \end{cases} \tag{1.3}$$

From (1.2)–(1.3) and denoting

$$\kappa(x) = 1 - g_0 a(x), \quad x \in \Omega, \tag{1.4}$$

we can rewrite problem (1.1) as the autonomous system

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