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Strong solutions to a parabolic equation with linear growth with respect to the gradient variable

Salvador Moll [a,*], Flavia Smarrazzo [b]

[a] *Departament d'Anàlisi Matemàtica, Universitat de València, Valencia, Spain*
[b] *Università Campus Bio-Medico di Roma, Roma, Italy*

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Abstract

In this paper we prove existence and uniqueness of strong solutions to the homogeneous Neumann problem associated to a parabolic equation with linear growth with respect to the gradient variable. This equation is a generalization of the time-dependent minimal surface equation. Existence and regularity in time of the solution is proved by means of a suitable pseudoparabolic relaxed approximation of the equation and a passage to the limit.

MSC: 35K93; 35K55; 35K67

Keywords: Minimal surface equation; Strong solutions; Pseudoparabolic regularization

1. Introduction

Let $\Omega \subset \mathbb{R}^N$ be a bounded domain with smooth boundary $\partial\Omega$. In this paper we consider the Neumann initial-boundary value problem

* Corresponding author.
E-mail addresses: j.salvador.moll@uv.es (S. Moll), flavia.smarrazzo@gmail.com (F. Smarrazzo).

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$$\begin{cases} u_t = \operatorname{div}(\mathbf{a}(u, \nabla u)) & \text{in } Q_T = (0,T) \times \Omega \\ \nu \cdot \mathbf{a}(u, \nabla u) = 0 & \text{on } S_T = (0,T) \times \partial\Omega \\ u = u_0 & \text{in } \{0\} \times \Omega\,, \end{cases} \tag{P}$$

where $u_0 \in L^2(\Omega)$ and ν is the unit outward normal on $\partial\Omega$.

Unless otherwise specified, the following assumptions will be made throughout the paper:

(H_1) the function $\mathbf{a} : \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}^N$ is Lipschitz continuous, $\mathbf{a}(z, 0) = 0$ and

$$|\mathbf{a}(z, \xi)| \le 1 \quad \text{for all } (z, \xi)\,. \tag{1.1}$$

Moreover, there exists $f : \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$, $f \in C^1(\mathbb{R} \times \mathbb{R}^N)$, convex with respect to ξ, such that

$$|\partial_z f(z, \xi)| \le \beta \quad \text{for all } (z, \xi) \in \mathbb{R} \times \mathbb{R}^N\,, \tag{1.2}$$

and

$$\mathbf{a}(z, \xi) = \nabla_\xi f(z, \xi) \quad \text{for all } (z, \xi) \in \mathbb{R} \times \mathbb{R}^N\,.$$

(H_2) There exist positive constants C_0, D_0 and C_1 such that

$$C_0|\xi| - D_0 \le f(z, \xi) \le C_1\big(|\xi| + |z| + 1\big) \quad \text{for every } (z, \xi) \in \mathbb{R} \times \mathbb{R}^N\,; \tag{1.3}$$

moreover, we also require

$$f^0(z, \xi) := \lim_{t \to 0^+} t f\left(z, \frac{\xi}{t}\right) = |\xi| \qquad (z \in \mathbb{R},\ \xi \in \mathbb{R}^N)\,. \tag{1.4}$$

Let us notice that, by the convexity of f,

$$\mathbf{a}(z, \xi) \cdot \xi \ge \mathbf{a}(z, \xi) \cdot \eta + f(z, \xi) - f(z, \eta) \tag{1.5}$$

and the following monotonicity condition

$$(\mathbf{a}(z, \xi) - \mathbf{a}(z, \eta)) \cdot (\xi - \eta) \ge 0 \tag{1.6}$$

holds true for any $z \in \mathbb{R}$ and $\xi, \eta \in \mathbb{R}^N$.

In the sequel we shall consider the function $h : \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$,

$$h(z, \xi) := \mathbf{a}(z, \xi) \cdot \xi\,. \tag{1.7}$$

It is easy to see that $h(z, \xi) \ge 0$ for all $z \in \mathbb{R}$ $\xi \in \mathbb{R}^N$. In addition, by (1.1) and (1.5) we also get

$$f(z, \xi) - f(z, 0) \le h(z, \xi) \le |\xi|\,, \tag{1.8}$$

whence

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