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Diffusive wave in the low Mach limit for non-viscous and heat-conductive gas ☆

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Abstract

The low Mach number limit for one-dimensional non-isentropic compressible Navier–Stokes system without viscosity is investigated, where the density and temperature have different asymptotic states at far fields. It is proved that the solution of the system converges to a nonlinear diffusion wave globally in time as Mach number goes to zero. It is remarked that the velocity of diffusion wave is proportional with the variation of temperature. Furthermore, it is shown that the solution of compressible Navier–Stokes system also has the same phenomenon when Mach number is suitably small.

Keywords: Compressible Navier–Stokes equations; Low Mach limit; Nonlinear diffusion wave

1. Introduction

The one-dimensional compressible Navier–Stokes system in Lagrangian coordinates reads

$$\begin{aligned} &v_t - u_x = 0, \\ &u_t + P_x = (\mu \frac{u_x}{v})_x, \\ &(e + \frac{u^2}{2})_t + (Pu)_x = (\kappa \frac{\theta_x}{v} + \mu \frac{u u_x}{v})_x, \end{aligned} \tag{1.1}$$

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where the unknown functions v, u and θ represent the specific volume, velocity and temperature, respectively, while $\mu > 0$ and $\kappa > 0$ denote the viscosity and heat conductivity coefficients respectively. Here, we consider the perfect gas, so that the pressure function P and the internal function e are given by

$$P = R\frac{\theta}{v}, \quad \text{and} \quad e = c_v\theta + const.,$$

where the parameters $R > 0$ and $c_v > 0$ are the gas constant and heat capacity at the constant volume respectively. For simplicity, we assume μ and κ are constants, and normalize $R = 1$ and $c_v = 1$.

The low Mach limit is an important and interesting problem in fluid dynamics. The first result is due to Klainerman and Majda [16,17], in which they proved the incompressible limit of the isentropic Euler equations to the incompressible Euler equations for local smooth solutions. In [1], Alazard showed the incompressible limit for Navier–Stokes equations in the whole space. Note that in [1], the solutions have the same states at the far fields. For related results, see [5,18,20,21,24,25,30] for Euler equations, and [6,9,12,15,22,28,29] for N–S equations.

Recently, Huang et al. [13] began to study the case that the solutions have different end states and found that the solutions of compressible Navier–Stokes equations converge to a nonlinear diffusion wave solution globally in time as Mach number goes to zero, which is related to the thermal creep flow [8]. That is, the flow in diffusion wave is only driven by the variation of temperature. This phenomenon is quite different from the constant case. Since the diffusion wave is independent of the viscosity $\mu = 0$, we conjecture that the result of [13] is still valid without viscosity, that is $\mu = 0$ in the system (1.1). Precisely speaking, we consider the non-viscous and heat-conductive gas in the following system

$$\begin{aligned} &v_t - u_x = 0, \\ &u_t + P_x = 0, \\ &(\theta + \frac{u^2}{2})_t + (Pu)_x = (\kappa\frac{\theta_x}{v})_x, \end{aligned} \tag{1.2}$$

where the only difference with the system (1.1) is that $\mu = 0$. We will prove that the solution of system (1.2) converges to a nonlinear diffusion wave solution globally in time as Mach number tends to zero. Moreover, as the Mach number is suitably small, the flow is only driven by the variation of temperature.

Let ε be the compressibility parameter, which represents the maximum Mach number of the fluid. As in [26], we set

$$t \to \varepsilon t, \quad x \to x, \quad u \to \varepsilon u, \quad \mu \to \varepsilon\mu, \quad \kappa \to \varepsilon\kappa.$$

By the above changes of variables, system (1.2) is written as

$$\begin{aligned} &v_t^\varepsilon - u_x^\varepsilon = 0, \\ &u_t^\varepsilon + \frac{1}{\varepsilon^2}P_x^\varepsilon = 0, \\ &[\theta^\varepsilon + \frac{1}{2}(\varepsilon u^\varepsilon)^2]_t + (P^\varepsilon u^\varepsilon)_x = \kappa(\frac{\theta_x^\varepsilon}{v^\varepsilon})_x. \end{aligned} \tag{1.3}$$

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