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Quasi-periodic solutions to nonlinear beam equations on compact Lie groups with a multiplicative potential ☆

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Abstract

The goal of this work is to study the existence of quasi-periodic solutions to nonlinear beam equations with a multiplicative potential. The nonlinearity is required to only finitely differentiable and the frequency is along a pre-assigned direction. The result holds on any compact Lie group or homogeneous manifold with respect to a compact Lie group, which includes standard torus $\mathbb{T}^d$, special orthogonal group $SO(d)$, special unitary group $SU(d)$, spheres $\mathbb{S}^d$ and the real and complex Grassmannians. The proof is based on a differentiable Nash–Moser iteration scheme.

MSC: 53C30; 35R03; 35B15

Keywords: Beam equations; Compact Lie groups; Multiplicative potential; Quasi-periodic solutions; Nash–Moser iteration

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1. Introduction

This paper concerns the existence of quasi-periodic solutions for forced nonlinear beam equations

$$u_{tt} + \Delta^2 u + V(\boldsymbol{x})u = \epsilon f(\omega t, \boldsymbol{x}, u), \quad \boldsymbol{x} \in \boldsymbol{M}, \tag{1.1}$$

where $\boldsymbol{M}$ is any simply connected compact Lie group with dimension d and rank r, $\epsilon > 0$ is a small number, $V \in C^q(\boldsymbol{M}; \mathbb{R})$ and $f \in C^q(\mathbb{T}^\nu \times \boldsymbol{M} \times \mathbb{R}; \mathbb{R})$ with q large enough. Assume the frequency vector $\omega \in \mathbb{R}^\nu$ with

$$\omega = \lambda\omega_0, \quad \lambda \in \Lambda := [1/2, 3/2], \quad |\omega_0| := \max_{1\le k\le\nu} |\omega_{0,k}| \le 1, \tag{1.2}$$

and that for some $\gamma_0 > 0$, the Diophantine condition holds:

$$|\omega_0 \cdot l| \ge 2\gamma_0 |l|^{-\nu}, \quad \forall l \in \mathbb{Z}^\nu \backslash \{0\}, \tag{1.3}$$

where $|l| := \max_{1\le k\le\nu} |l_k|$. Moreover we suppose

$$\Delta^2 + V(\boldsymbol{x}) \ge \kappa_0 \mathrm{I} \quad \text{with } \kappa_0 > 0. \tag{1.4}$$

Equation (1.1) is interesting by itself. It is derived from the following Euler–Bernoulli beam equation

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(EI\frac{\mathrm{d}^2 u}{\mathrm{d}x^2}\right) = g,$$

which describes the relationship between the applied load and the beam's deflection, where the curve $u(x)$ describes the deflection of the beam at some position x in the z direction. Moreover g is distributed load which may be a function of x, u or other variables; I is the second moment of area of the beam's cross-section; E is the elastic modulus; the product EI is the flexural rigidity. Derivatives of the deflection u have significant physical significance: u_x is the slope of the beam; $-EIu_{xx}$ is the bending moment of the beam and $-(EIu_{xx})_x$ is the shear force of the beam. The dynamic beam equation is the Euler–Lagrange equation

$$\mu\frac{\partial^2 u}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 u}{\partial x^2}\right) = g, \tag{1.5}$$

where μ is the mass per unit length. If E and I are independent of x, then equation (1.5) can be reduced to $\mu u_{tt} + EIu_{xxxx} = g$. After a time rescaling $t \to ct$ with $c = \sqrt{EI/\mu}$, we can obtain

$$u_{tt} + u_{xxxx} = \tilde{g} \quad \text{with } \tilde{g} = \frac{\mu}{EI} g.$$

The search for periodic or quasi-periodic solutions to nonlinear PDEs has a long standing tradition. There are two main approaches: one is the infinite-dimensional KAM (Kolmogorov–Arnold–Moser) theory to Hamiltonian PDEs, refer to Kuksin [22], Wayne [28], Pöschel [23],

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