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On a local energy decay estimate of solutions to the hyperbolic type Stokes equations

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Abstract

In this paper, we discuss a local energy decay estimate of solutions to the initial-boundary value problem for the hyperbolic type Stokes equations of incompressible fluid flow in an exterior domain and a perturbed half-space. The equations are linearized version of the hyperbolic Navier–Stokes equations introduced by Racke and Saal [15], which are obtained as a delayed case for the deformation tensor in the incompressible Navier–Stokes equations. Our proof of the local energy decay estimate is based on Dan and Shibata [2]. In [2], they treated the dissipative wave equations in an exterior domain and discussed the local energy decay estimate. Our approach uses the fact that applying the Helmholtz projection to the hyperbolic type Stokes equations, we obtain equations similar to the dissipative wave ones.

Keywords: Hyperbolic Navier Stokes equations; Local energy decay estimate; Exterior domain; Perturbed half-space

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1. Introduction

Let Ω be a smooth domain in the n-dimensional Euclidean space $\mathbf{R}^n$ ($n \geq 2$). We consider a motion of incompressible fluid flow occupying Ω which satisfies the initial-boundary value problem:

$$\begin{cases} \tau u_{tt} - \nu\Delta u + u_t + \nabla\pi + \tau\nabla\pi_t = -\tau(u\cdot\nabla)u_t - ((\tau u_t + u)\cdot\nabla)u & \text{in } \Omega\times(0,\infty), \\ \nabla\cdot u = 0 & \text{in } \Omega\times(0,\infty), \\ u|_{\partial\Omega} = 0, \quad (u, u_t)|_{t=0} = (u_0, u_1) \end{cases} \tag{1.1}$$

with unknown vector valued function $u = u(x,t) = (u_1(x,t), \dots, u_n(x,t))$ and unknown scalar valued function $\pi = \pi(x,t)$ describing the velocity field and the pressure respectively, where $x = (x_1, \dots, x_n)$ denotes a spatial point of Ω and t is a time variable. Moreover, $\partial\Omega$ is the boundary of Ω, (u_0, u_1) is a given initial data and $\nu > 0$ and $\tau > 0$ denote the viscosity coefficient and the relaxation parameter satisfying $\tau < 1$ respectively. Here and hereafter, we write

$$\partial_t^m = \left(\frac{\partial}{\partial t}\right)^m \quad (m \in \mathbf{N}\cup\{0\}), \quad \partial_t = \partial_t^1, \quad (u_t, u_{tt}, \pi_t) = \left(\frac{\partial u}{\partial t}, \frac{\partial^2 u}{\partial t^2}, \frac{\partial \pi}{\partial t}\right),$$

$$\partial_j^k = \frac{\partial^k}{\partial x_j^k} \quad (j = 1, \dots, n,\ k \in \mathbf{N}), \quad \partial_j = \partial_j^1, \quad \Delta = \sum_{j=1}^n \partial_j^2, \quad \Delta u = (\Delta u_1, \dots, \Delta u_n),$$

$$\nabla\pi = (\partial_1\pi, \dots, \partial_n\pi), \quad \nabla\cdot u = \sum_{j=1}^n \partial_j u_j, \quad w\cdot\nabla = \sum_{j=1}^n w_j\partial_j,$$

where $w = (w_1, \dots, w_n)$. The equations (1.1) can be derived from the classical Navier–Stokes equations as follows. The classical ones determined by Fourier type law are represented by

$$\begin{cases} u_t + (u\cdot\nabla)u + \nabla\pi = \operatorname{Div} 2S, \quad \nabla\cdot u = 0 \quad \text{in } \Omega\times(0,\infty), \\ u|_{\partial\Omega} = 0, \quad u|_{t=0} = u_0, \end{cases} \tag{1.2}$$

where the deformation tensor $S = (S_{jk})_{j,k=1}^n$ and $\operatorname{Div} S$ are given by

$$S_{jk} = \frac{\nu}{2}(\partial_j u_k + \partial_k u_j) \quad \text{and} \quad \operatorname{Div} S = \Big(\sum_{j=1}^n \partial_j S_{jk}\Big)_{k=1}^n$$

respectively. In this case, the divergence free condition $\nabla\cdot u = 0$ implies

$$\operatorname{Div} 2S = \nu\Delta u.$$

On the other hand, Cattaneo type law:

$$S + \tau\partial_t S = \frac{\nu}{2}(\partial_j u_k + \partial_k u_j)_{j,k=1}^n \tag{1.3}$$