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Differential equations driven by rough paths with jumps

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Abstract

We develop the rough path counterpart of Itô stochastic integration and differential equations driven by general semimartingales. This significantly enlarges the classes of (Itô/forward) stochastic differential equations treatable with pathwise methods. A number of applications are discussed.

MSC: primary 60H99; secondary 60H10

Keywords: Rough paths with jumps; Itô stochastic and rough differential equations with jumps; General semimartingales; Limit theorems

Contents

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0. Introduction and notation

In many areas of engineering, finance and mathematics one encounters equations of the form

$$dy_t = f(y_t)dx_t \ , \tag{1}$$

where x is a multi-dimensional driving signal, f a collection of nice driving vector fields.[1] For $x \in C^1$, this can be written as time-inhomogenous ODE of the form $\dot{y}(t) = f(y(t))\dot{x}(t)$ and there is no ambiguity in its interpretation. This is still the case for rectifiable drivers,

$$x \in C^{1-var} \equiv C \cap V^1,$$

i.e. continuous paths of locally finite 1-variation, say on $[0, T]$, in which case there is perfect meaning to the (Riemann–Stieltjes) integral equation

[1] At the price of replacing f by (f_0, f) and x by (t, x) formulation (1) immediately allows for a drift term.