ARTICLE IN PRESS YJDEQ:9190

Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Complicated asymptotic behavior of solutions for porous medium equation in unbounded space ☆

Liangwei Wang [a,*], Jingxue Yin [b], Yong Zhou [c]

[a] *Key Laboratory for Nonlinear Science and System Structure, College of Mathematics and Statistics, Chongqing Three Gorges University, Chongqing, 404100, China*

[b] *School of Mathematical Science, South China Normal University, Guangzhou, 510631, China*

[c] *School of Mathematics (Zhuhai), Sun Yat-Sen University, Zhuhai, 519082, China*

Received 8 January 2017; revised 13 August 2017

Abstract

In this paper, we find that the unbounded spaces $Y_\sigma(\mathbb{R}^N)$ $(0 < \sigma < \frac{2}{m-1})$ can provide the work spaces where complicated asymptotic behavior appears in the solutions of the Cauchy problem of the porous medium equation. To overcome the difficulties caused by the nonlinearity of the equation and the unbounded solutions, we establish the propagation estimates, the growth estimates and the weighted L^1–L^∞ estimates for the solutions.

MSC: 35K55; 35B40

Keywords: Asymptotic behavior; Porous medium equation; Unbounded function; Propagation estimates; Growth estimates; Weighted L^1–L^∞ estimates

☆ This research was supported by NSFC (11601047 and 11771156), NSF of CQ (cstc2016jcyjA0596), Innovation Team Building at Institutions of Higher Education in Chongqing (CXTDX201601035), Chongqing Municipal Key Laboratory of Institutions of Higher Education (Grant No. [2017]3) and Research project of Chongqing Three Gorges University (17ZP13).

* Corresponding author.

E-mail addresses: wanglw08@163.com (L. Wang), yjx@scnu.edu.cn (J. Yin), zhouyong3@mail.sysu.edu.cn (Y. Zhou).

https://doi.org/10.1016/j.jde.2018.01.037

1. Introduction

In this paper, we consider the complicated asymptotic behavior of solutions for the Cauchy problem of the porous medium equation

$$\frac{\partial u}{\partial t} - \Delta u^m = 0 \qquad \text{in} \quad \mathbb{R}^N \times (0,\infty), \tag{1.1}$$

$$u(x,0) = u_0(x) \qquad \text{in} \quad \mathbb{R}^N, \tag{1.2}$$

where $m > 1$ and the nonnegative initial data $u_0 \in Y_\sigma(\mathbb{R}^N) \equiv \{\varphi \in C(\mathbb{R}^N) : \lim_{|x|\to\infty}(1+|x|^2)^{-\frac{\sigma}{2}}\varphi(x) = 0\}$.

For solutions of some evolution equations, the complicated asymptotic behavior occurring or not mainly depends on the work spaces where the initial data belong to, see [1–6]. Consider the problem (1.1)–(1.2). When the nonnegative initial data $u_0 \in L^1(\mathbb{R}^N)$, it is well-known that the solutions $u(x,t)$ converge to the Barenblatt solution U_M in $L^1(\mathbb{R}^N)$ as $t \to \infty$ [7]. While if $u_0 \in L^\infty(\mathbb{R}^N)$, it was first found in 2002 by Vázquez and Zuazua that the bounded space $L^\infty(\mathbb{R}^N)$ can provide the work space where complicated asymptotic behavior of rescaled solutions $u(t^{\frac{1}{2}}x,t)$ may take place [8]. In our previous papers [9–12], we proved that the rescaled solutions $t^\mu u(t^\beta x,t)$ $(\mu,\beta > 0)$ for the problem (1.1)–(1.2) may present complicated asymptotic behavior in the bounded space $C_0(\mathbb{R}^N)$ by using the L^1–L^∞ estimates [3] and the finite propagation properties [13] for the bounded solutions of the problem (1.1)–(1.2) with the initial data $u_0 \in C_0(\mathbb{R}^N)$. Since 2003, Cazenave, Dickstein and Weissler [14–17] gave a series of results about the complicated asymptotic behavior of solutions for the Cauchy problem of heat equation in some bounded spaces by using the integral expressions of solutions, the linearity of equation and some well-known estimates about the solutions. For other evolution equations, the complicated asymptotic behavior of solutions in some bounded spaces had also been investigated in many papers, see [18–21].

Note that the problems about the complicated asymptotic behavior of solutions in the above works are only considered in some bounded spaces. It follows from the existence theory for the porous medium equations [22,23] that the solutions of the problem (1.1)–(1.2) are global even if the initial data belong to some unbounded spaces. Hence the complicated asymptotic behavior of solutions for the porous medium equations may occur in such unbounded spaces. Our interest here is to investigate the complicated asymptotic behavior of solutions for the problem (1.1)–(1.2) in the unbounded spaces $Y_\sigma(\mathbb{R}^N)$ with $0 < \sigma < \frac{2}{m-1}$. These spaces can contain functions with polynomial growth. So, the solutions $u(x,t)$ of (1.1)–(1.2) with initial data $u_0 \in Y_\sigma(\mathbb{R}^N)$ may be unbounded solutions. To overcome the difficulties caused by the unbounded solutions and the nonlinearity of equation (1.1), we need to estimate the propagation rate and the growth rate for the solutions in some weighted spaces, and give the weighted L^1–L^∞ estimates for the solutions by following the well-known Moser's ideas which had been successfully used to prove the existence of weak solutions for the problem (1.1)–(1.2) [22]. Form these, we can get that there exists a function $\phi \in Y_\sigma(\mathbb{R}^N)$ such that the ω-limit set $\omega_\sigma^{\mu,\beta}(u_0) = Y_\sigma^+(\mathbb{R}^N)$ for some μ, β, where $\omega_\sigma^{\mu,\beta}(u_0) \equiv \{f \in Y_\sigma(\mathbb{R}^N);\ \exists t_n \to \infty \text{ s.t. } t_n^{\frac{\mu}{2}} u(t_n^\beta \cdot,\ t_n) \xrightarrow{t_n\to\infty} f \text{ in } Y_\sigma(\mathbb{R}^N)\}$, $Y_\sigma^+(\mathbb{R}^N) \equiv \{\varphi \in Y_\sigma(\mathbb{R}^N);\ \varphi \ge 0 \text{ and } \varphi(0) = 0\}$ and $u(x,t)$ is the solution of the problem (1.1)–(1.2) with the initial data $u_0(x) = \phi(x)$. So the unbounded spaces $Y_\sigma(\mathbb{R}^N)$ can provide the work spaces where

Please cite this article in press as: L. Wang et al., Complicated asymptotic behavior of solutions for porous medium equation in unbounded space, J. Differential Equations (2018), https://doi.org/10.1016/j.jde.2018.01.037