ARTICLE IN PRESS YJDEQ:9170

Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Bounding the number of limit cycles of discontinuous differential systems by using Picard–Fuchs equations

Jihua Yang [a,b], Liqin Zhao [a,*,1]

[a] *School of Mathematical Sciences, Beijing Normal University, Laboratory of Mathematics and Complex Systems, Ministry of Education, Beijing 100875, People's Republic of China*

[b] *School of Mathematics and Computer Science, Ningxia Normal University, Guyuan 756000, People's Republic of China*

Received 15 September 2017; revised 5 January 2018

Abstract

In this paper, by using Picard–Fuchs equations and Chebyshev criterion, we study the upper bounds of the number of limit cycles given by the first order Melnikov function for discontinuous differential systems, which can bifurcate from the periodic orbits of quadratic reversible centers of genus one (r19): $\dot{x} = y - 12x^2 + 16y^2$, $\dot{y} = -x - 16xy$, and (r20): $\dot{x} = y + 4x^2$, $\dot{y} = -x + 16xy$, and the periodic orbits of the quadratic isochronous centers $(S_1): \ \dot{x} = -y + x^2 - y^2$, $\dot{y} = x + 2xy$, and $(S_2): \ \dot{x} = -y + x^2$, $\dot{y} = x + xy$. The systems (r19) and (r20) are perturbed inside the class of polynomial differential systems of degree n and the system (S_1) and (S_2) are perturbed inside the class of quadratic polynomial differential systems. The discontinuity is the line $y = 0$. It is proved that the upper bounds of the number of limit cycles for systems (r19) and (r20) are respectively $4n - 3 (n \geq 4)$ and $4n + 3 (n \geq 3)$ counting the multiplicity, and the maximum numbers of limit cycles bifurcating from the period annuluses of the isochronous centers (S_1) and (S_2) are exactly 5 and 6 (counting the multiplicity) on each period annulus respectively.

Keywords: Limit cycle; Discontinuous differential system; Picard–Fuchs equation; Piecewise smooth differential system

* Corresponding author.
E-mail addresses: yangjh@mail.bnu.edu.cn (J. Yang), zhaoliqin@bnu.edu.cn (L. Zhao).
1 Supported by National Natural Science Foundation of China (11671040, 11701306).

https://doi.org/10.1016/j.jde.2018.01.017

1. Introduction and the main results

In these last years a great interest has appeared for studying the number of limit cycles of discontinuous differential systems, that is differential equations with discontinuous right-hand sides. This interest has been stimulated by discontinuous phenomena in control systems [1], impact and friction mechanics [2], nonlinear oscillations [16], economics [10], and biology [11], and it has become certainly one of the common frontiers between mathematics, physics and engineering.

It is known that Loud [14] proved that after an affine change of variables and a rescaling of the independent variable any quadratic isochronous center can be written as one of the following systems:

$$(S_1):\ \dot{x}=-y+x^2-y^2,\ \dot{y}=x+2xy,$$
$$(S_2):\ \dot{x}=-y+x^2,\ \dot{y}=x+xy,$$
$$(S_3):\ \dot{x}=-y-\frac{4}{3}y^2,\ \dot{y}=x-\frac{16}{3}xy,$$
$$(S_4):\ \dot{x}=-y+\frac{16}{3}x^2-\frac{4}{3}y^2,\ \dot{y}=x+\frac{8}{3}xy.$$

Chicone and Jacobs [3] proved that, under all quadratic polynomial perturbations, at most 1 and 2 limit cycles bifurcate from the period annuluses of (S_1) and (S_2) respectively. By the averaging method of first order, Llibre and Mereu [13] studied the number of limit cycles bifurcating from the period annuluses of quadratic isochronous centers (S_1) and (S_2) when they are perturbed inside a class of piecewise smooth quadratic polynomial differential systems. They found that at least 4 and 5 limit cycles can bifurcate from the period annuluses of (S_1) and (S_2) respectively, and both of which show that the piecewise smooth systems have more limit cycles than the smooth ones.

Recently, by average method, Han [7] studied the maximum number of periodic solutions of piecewise smooth periodic equations.

According to the classification provided by [4], there are essentially 22 types of quadratic reversible centers of genus one, namely $(r1)$–$(r22)$. For

$$(r19):\ \dot{x}=y-12x^2+16y^2,\ \dot{y}=-x-16xy,$$
$$(r20):\ \dot{x}=y+4x^2,\ \dot{y}=-x+16xy,$$

Hong etc. [8], [9] proved that, under all polynomial perturbations of degree n, the maximum numbers of zeros of the first order Melnikov functions are $6n-12$ if $n\geq 4$ and $6n-9$ if $n\geq 3$ respectively.

Motivated by [7], [8], [9] and [13], in the present paper, we first study the upper bound of the number of limit cycles bifurcating from the period annuluses of $(r19)$ and $(r20)$ when they are perturbed inside any discontinuous polynomial differential systems of degree n, and then we obtain the sharp upper bounds of the number of limit cycles bifurcating from the period annuluses of quadratic isochronous centers (S_1) and (S_2) when they are perturbed inside any discontinuous quadratic polynomial differential systems.

The main results are follows.

Please cite this article in press as: J. Yang, L. Zhao, Bounding the number of limit cycles of discontinuous differential systems by using Picard–Fuchs equations, J. Differential Equations (2018), https://doi.org/10.1016/j.jde.2018.01.017