ARTICLE IN PRESS YJDEQ:9173

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J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

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Curved fronts in the Belousov–Zhabotinskii reaction–diffusion systems in $\mathbb{R}^2$

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Received 27 April 2017; revised 9 November 2017

Abstract

In this paper we consider a diffusion system with the Belousov–Zhabotinskii (BZ for short) chemical reaction. Following Brazhnik and Tyson [4] and Pérez-Muñuzuri et al. [45], who predicted V-shaped fronts theoretically and discovered V-shaped fronts by experiments respectively, we give a rigorous mathematical proof of their results. We establish the existence of V-shaped traveling fronts in $\mathbb{R}^2$ by constructing a proper supersolution and a subsolution. Furthermore, we establish the stability of the V-shaped front in $\mathbb{R}^2$.

MSC: 35K40; 35K57; 35C07; 35B35; 35B40

Keywords: Belousov–Zhabotinskii reaction; Traveling waves; Curved fronts; Supersolution and subsolution; Comparison principle

1. Introduction

In this paper, we study the following reaction–diffusion system

$$\begin{cases} u_t(\mathbf{x},t) = \Delta u(\mathbf{x},t) + u(\mathbf{x},t)(1 - u(\mathbf{x},t) - rv(\mathbf{x},t)) \\ v_t(\mathbf{x},t) = \Delta v(\mathbf{x},t) - bu(\mathbf{x},t)v(\mathbf{x},t) \end{cases} \tag{1.1}$$

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https://doi.org/10.1016/j.jde.2018.01.020

0022-0396/

Please cite this article in press as: H.-T. Niu et al., Curved fronts in the Belousov–Zhabotinskii reaction–diffusion systems in $\mathbb{R}^2$, J. Differential Equations (2018), https://doi.org/10.1016/j.jde.2018.01.020

where $r > 0, b > 0$ are parameters. Note that in system (1.1) r is a key parameter which decides the characteristic of the BZ system. Precisely, the BZ system is monostable if $r \in (0, 1]$ while it is bistable if $r > 1$. But the monostability/bistability here for the BZ system is not the standard definition. See [42] and references therein for more details. In this paper we consider the case $r > 1$.

The BZ chemical reaction was discovered in 1959 by Belousov [1]. Belousov discovered that the oxidation of citric acid by bromate in dilute sulfuric acid, in the presence of cerium ions as catalyst-indicator, proceeds in a curious fashion: the cerium ions oscillate between different oxidation states, causing the solution to change color periodically from clear to pale yellow back to clear. In 1970, Zaikin and Zhabotinskii [57] found that traveling waves of chemical activity may form and propagate through the medium when the reagent is spread in a thin layer on a flat surface such as a peri dish. In order to describe the phenomena discovered in [57], Field et al. [7] proposed a complex system as a model for it, which was simplified by Field and Noyes [8]. Later in 1977, based on experimental and numerical results, Murray [29,30] introduced an artificial parameter r and nondimensionalized the model of Field and Noyes to be system (1.1), the front solution of which provides an appropriate mathematical tool for the description of planar waves observed in [57].

System (1.1) has attracted a lot of attention since then. Gibbs [11] established the existence of a traveling wave for (1.1) in the absence of the diffusion term in the first or second equation. In [46], the existence of traveling waves was established for an interesting class of locally monotone systems by means of fixed-point theorem. Troy [44] and Kanel [22] studied the existence as well as the wave speed of traveling fronts of system (1.1). Then Klaasen and Troy [25] investigated the asymptotic behavior of the solutions of (1.1). Kapel [23] proved the existence of traveling fronts of (1.1) and improved the accuracy of estimate for admissible wave speed. Trofimchuk et al. [42] studied the existence and asymptotic behaviors of traveling fronts of system (1.1), including the case with time delay. For numerical studies of system (1.1), we refer the readers to [30,31,35]. More references are referred to [27,43,52,55,56].

In 1995, by using kinematic approach, Brazhnik and Davydov [3] predicted that V-shaped structures exist and are stable in chemical excitable media with a BZ reaction. Soon, their theoretical prediction was convinced by Pérez-Muñuzuri et al. [45] in a BZ chemical reaction in experiment. Pérez-Muñuzuri et al. observed V-shaped stable patterns moving with a constant shape and stable velocity in experiments, which is in good agreement with the theoretical predictions in [3]. They found that along all the front length, the curvature of the V-shaped front is equal to zero except at the vertex. Thus the V-shaped wave front can be seen as a combination of two planar waves with speed c. If the speed of the V-shaped front as a whole is s, then

$$s = \frac{c}{\sin\alpha},$$

where 2α is the inner angle of the V-shaped front. This result was shown by theoretical prediction in [3] as well as in [45] by numerical simulation. While working on the flame propagation of Bunsen burners, Hamel and his coworkers [2,16,17] also derived the same relationship between the speed of planar waves and curved waves. That is to say, the curvature effect accelerates the propagation speed. However, there is still no rigorous mathematical results about curved fronts of systems (1.1) as far as we know. The aim of this paper is to give an rigorous mathematical proof of the existence and stability of V-shaped fronts in the BZ system.

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