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YJDEQ:9174

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J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Classification and evolution of bifurcation curves for the one-dimensional Minkowski-curvature problem and its applications

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Received 16 August 2017; revised 2 January 2018

Abstract

In this paper, we study the classification and evolution of bifurcation curves of positive solutions for the one-dimensional Minkowski-curvature problem

$$\begin{cases} -\left(u'/\sqrt{1-u'^2}\right)' = \lambda f(u), & \text{in } (-L, L), \\ u(-L) = u(L) = 0, \end{cases}$$

where $\lambda, L > 0$, $f \in C[0,\infty) \cap C^2(0,\infty)$ and $f(u) > 0$ for $u \geq 0$. Furthermore, we show that, for sufficiently large $L > 0$, the bifurcation curve S_L may have arbitrarily many turning points. Finally, we apply these results to obtain the global bifurcation diagrams for *Ambrosetti–Brezis–Cerami problem, Liouville–Bratu–Gelfand problem* and *perturbed Gelfand problem* with the Minkowski-curvature operator, respectively. Moreover, we will make two lists which show the different properties of bifurcation curves for Minkowski-curvature problems, corresponding semilinear problems and corresponding prescribed curvature problems.

MSC: 34B15; 34B18; 34C23; 74G35

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https://doi.org/10.1016/j.jde.2018.01.021

1. Introduction

In this paper, we study the bifurcation curve of positive solutions for the one-dimensional Minkowski-curvature problem

$$\begin{cases} -\left(u'/\sqrt{1-u'^2}\right)' = \lambda f(u), & \text{in } (-L, L), \\ u(-L) = u(L) = 0, \end{cases} \tag{1.1}$$

where $\lambda, L > 0$, $f \in C[0, \infty) \cap C^2(0, \infty)$ and $f(u) > 0$ for $u > 0$. The problem (1.1) plays an important role in certain fundamental issues in differential geometry and in the special theory of relativity, see for example [7,10]. We refer the readers, for motivations and results, to [2] and the references cited therein.

For any fixed $L > 0$, we define the bifurcation curve S_L of (1.1) on the $(\lambda, \|u\|_\infty)$-plane by

$$S_L = \left\{(\lambda, \|u_\lambda\|_\infty) : \lambda > 0 \text{ and } u_\lambda \in C^2(-L, L) \cap C[-L, L] \right. \\ \left. \text{is a positive solution of (1.1)}\right\}. \tag{1.2}$$

For comparison, we mention two similar problems. First, consider the semilinear problem

$$\begin{cases} -u'' = \lambda f(u), & \text{in } (-L, L), \\ u(-L) = u(L) = 0. \end{cases} \tag{1.3}$$

In (1.3), the constant L can be scaled out so one defines the bifurcation curve $\bar{S}$ of (1.3) on the $(\lambda, \|u\|_\infty)$-plane by

$$\bar{S} \equiv \{(\lambda, \|u_\lambda\|_\infty) : \lambda > 0 \text{ and } u_\lambda \text{ is a positive solution of (1.3)}\}.$$

Second, consider the quasilinear prescribed curvature problem (in the Euclidean space)

$$\begin{cases} -\left(u'/\sqrt{1+u'^2}\right)' = \lambda f(u), & \text{in } (-L, L), \\ u(-L) = u(L) = 0. \end{cases} \tag{1.4}$$

For any fixed $L > 0$, we define the bifurcation curve $\tilde{S}_L$ of (1.4) on the $(\lambda, \|u\|_\infty)$-plane by

$$\tilde{S}_L = \left\{(\lambda, \|u_\lambda\|_\infty) : \lambda > 0 \text{ and } u_\lambda \in C^2(-L, L) \cap C[-L, L] \right. \\ \left. \text{is a positive solution of (1.4)}\right\}.$$

The shapes of the bifurcation curves provide many important clues to the qualitative properties of the problems (1.1), (1.3) and (1.4). For this reason, we first give some terminologies related to the shapes of bifurcation curves S_L on the $(\lambda, \|u\|_\infty)$-plane (while similar terminologies for $\bar{S}$ and $\tilde{S}_L$ also hold), see Fig. 1.1.

Please cite this article in press as: S.-Y. Huang, Classification and evolution of bifurcation curves for the one-dimensional Minkowski-curvature problem and its applications, J. Differential Equations (2018), https://doi.org/10.1016/j.jde.2018.01.021